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PLANE WAVES OF LIGHT

II. Reflection and Refraction

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§13. *Introduction:* In a previous paper[1] the propagation of plane light waves through a homogeneous isotropic medium was considered from the standpoint of Maxwell's theory. It was shown, principally, that there are two *essentially* different types of plane waves, the one being plane-polarized light, and the other a type which, for lack of a recognized name, we called "hybrid." All other waves that could be represented by an exponential of the type

$$e^{iq(\alpha x+\beta y+\gamma z)-ipt}$$

could be produced by the superposition of plane-polarized or hybrid waves: but neither of these could be derived from the superposition of waves of the other type. In that paper certain references were made to reflection and refraction: for instance, it was stated that the electromagnetic disturbance on the "dark" side of a totally reflecting glass boundary is "hybrid." But these statements were made without proof, and nowhere in the paper was the problem of two media discussed.

The present paper aims to present the equations governing reflection and refraction, and to show how they lead in metals to hybrid waves. As in the earlier paper, many of the results are well known, but I have failed to find them collected in one place in a systematic manner. Besides, the classical notation for reflection and refraction proves to be a very awkward medium of expression when angles of refraction become "imaginary," as they do in dealing with metals, and it appears desirable

[1] Plane Waves of Light: I. Electromagnetic Behavior, J.O.S.A. & R.S.I., *15*, p. 137; 1927.

to have a symbolism better suited to our purpose. It is for this reason that a certain amount of old material masquerades in a new dress in succeeding sections.

§14. *The Electromagnetic Boundary Conditions:* Why do we associate with an incident beam *two* others: a reflected one and a refracted one? And why do we say these are of the same frequency as the incident beam? One answer, of course, is that we observe this to be true; but in discussing such a matter as reflection from a purely theoretical standpoint such as that afforded by Maxwell's Theory it is interesting to see how far our theoretical hypotheses alone would carry us. With reference to these specific questions we shall find they carry us exactly as far as the experimental evidence goes, and (so long as we deal with dielectrics) no further.

To see this, we may recall that the same physical ideas which lead to Maxwell's equations also show that if two media are in contact, their electromagnetic fields must be related in a very definite fashion. In other words, they give, not only the equations themselves, but certain boundary conditions as well. These conditions are:

I. The normal component of μH must be continuous.

II. The tangential components of E must be continuous.

III. The normal components of ϵE must differ by an amount equal to the surface charge. That is, if there is a charge s per unit area, and if the normal to the surface is called z, the values on the positive and negative sides must satisfy the relation $\epsilon_+ E_{z+} - \epsilon_- E_{z-} = s$.

IV. The tangential components of H differ by amounts depending on the currents flowing in the surface. Taking the direction of z normal to the surface, and x and y tangential to it, the relations are

$$H_{y+} - H_{y-} = -I_x/c,$$
$$H_{x+} - H_{y-} = +I_y/c,$$

I_x and I_y denoting the currents flowing across unit lengths parallel to the x and y axes, respectively.

In the case of dielectrics, there is no occasion to expect the existence of surface charges or currents. Hence all four vector-components are continuous. But in the case of metals, conduction currents do give rise to surface charges, and probably also to surface current sheets.[2] The

[2] These sheets are of speculative character, since they depend upon the behavior of the free surface electricity under the influence of the electric field. It appears possible, however, that they may introduce very weak light of *twice the incident frequency*, which would be interesting, if true.

latter, however, if they exist at all, are so feeble that they may be ignored in this discussion. Hence we have *three* general conditions:

The normal component of μH, and the tangential components of E and H, are all continuous.

Obviously, then, if a wave of light is moving through the first medium, some sort of disturbance must exist in the second, and the mathematical problem presents itself of determining the character of the latter disturbance. But, of course, to do that presupposes that we know the nature of the disturbance in the first medium. In our study we shall assume it to be a wave of the exponential form[3]

$$\begin{aligned} E_x &= Ele^{iq(ax+by+cz)-ipt}, \\ &\cdots\cdots\cdots\cdots \\ H_x &= Hn'e^{iq(ax+by+cz)-ipt}. \end{aligned} \tag{2}$$

That is, it is plane, or elliptically polarized, or hybrid.

The simplest guess which we can make regarding the nature of the disturbance in the second medium is, that it is of the same nature as that in the first; that is, that it is of the form

$$\begin{aligned} E_{x_1} &= E_1 l_1 e^{iq_1(a_1x+b_1y+c_1z)-ip_1t}, \\ &\cdots\cdots\cdots\cdots \\ H_{x_1} &= H_1 n_1' e^{iq_1(a_1x+b_1y+c_1z)-ip_1t}. \end{aligned}$$

If so, at the bounding surface (which we assume to be the plane $z=0$) E_{x_1}, E_{y_1}, H_{x_1}, H_{y_1}, $\epsilon_1 E_{z_1}$ and $\mu_1 H_{z_1}$ must equal the corresponding quantities without the subscripts.

We take first the relation between E_x and E_{x_1}. It is

$$Ele^{iq(ax+by)-ipt} = E_1 l_1 e^{iq_1(a_1x+b_1y)-ip_1t}.$$

This relation must be true at every instant and at every point of the surface, which it will be only provided

$$\begin{aligned} p &= p_1, \\ qa &= q_1 a_1, \\ qb &= q_1 b_1. \end{aligned}$$

Physically, these equations state that the light is of the same frequency in both media, and that the traces made upon the interface by waves in either medium move with like velocities.

[3] Equations are numbered as in the previous paper.

Moreover, q_1 is uniquely determined by the properties of the medium, and therefore these equations completely determine a_1 and b_1. Then from the equation

$$a_1^2+b_1^2+c_1^2=1, \tag{3}$$

the triplet a_1, b_1, c_1 can be *completely* determined, except that c_1 is obtained by extracting a square root, and may be either positive or negative. This triplet determines the direction of propagation and the damping of the wave. Hence:

If it is possible to satisfy the boundary conditions at all by a wave of this form, it must move in one of two directions.

Moreover, whether these waves are of the plane-polarized or hybrid type, and in the latter event the direction and degree of the damping, are completely determined.

This latter statement is justified by the fact that, as was seen in the earlier paper, the nature of hybrid light is entirely determined by the triplet (a, b, c).

We must add that all this is quite independent of our assumption that no surface charges exist. The presence of such surface conditions may modify the plane of polarization or the intensity of the wave. They cannot affect its type. This is obvious from the fact that we have used only condition II, which is universally valid.

There are, then, just two sorts of waves which may exist in the second medium. Presumably either of these waves might exist alone, or both might exist together, and our next step must be to investigate whether the boundary conditions can be satisfied by either of these three possibilities. We need not carry out the mathematics here: the result is, that one beam alone can exist only provided a is given a special value; that is, provided the light in the first medium is incident at a particular angle, which is known as the polarizing angle. But the two together can satisfy the boundary values, no matter what the beam in the first medium may be.

Generally, therefore, there are two waves in one of our media. Mathematically, we have thought of these as arising from the known disturbance in the other medium: physically we ordinarily invert the roles, regard one of the pair (the "incident beam") as known, and say that it gives rise to both its mate (the "reflected beam") and the disturbance in the other medium (the "refracted beam"). Both are ways of thinking: the logical fact is that the three coexist. But from this point on we shall find the physical way the better.

Finally, we must not forget that all this has been achieved by *assuming* that all disturbances were of exponential type; and we must now inquire whether any *other* solution is possible. It is not difficult to see that if there is such a solution, it must be equivalent to superposing upon the waves already found, some disturbance which vanishes *over the entire boundary and at all times*. As such a disturbance could exist without any of the waves with which we are dealing, it cannot be said to be "produced by" any of them.[4] Hence, if such solutions exist at all, they are outside the range of our physical problem.

So much for pure logic. We now return to algebra.

§15. *The Equations for Simple Reflection at a Plane Boundary*: We have justified our belief in two facts: (a) That a wave of exponential form "gives rise to" waves of exponential form, and to such waves only, when it is allowed to impinge on a plane boundary. (b) That the boundary conditions can only be satisfied by assuming *three* waves: the "incident" and "reflected" beams in one medium, the "refracted" beam in the other. This leaves us, then, with only the problem of determining the exact relationships which exist between the various constants that characterize the three waves.

It will serve our purpose best to assume that the incident light is plane-polarized, with either the electric or the magnetic vector in the plane of incidence, for this will permit us to choose our coordinate axes in a very convenient fashion. Moreover, the loss in generality is not serious, for the incident beam is seldom hybrid in practice, and all other cases can be built up by superposing waves of the types discussed.

We choose our axes in such a way that the bounding plane contains x and y, and the plane of incidence x and z. We also agree to denote by the subscript 1 those quantities which appertain to the refracted beam, by 2 those associated with the reflected beam, while those associated with the incident beam are without subscripts. Finally, we adopt the letter r to denote the sine of the angle of incidence. Then the triplet a, b, c automatically takes the form r, 0, $-(1-r^2)^{1/2}$; while from the argument of §14 we conclude that

$$q_1 a_1 = q a,$$
$$q_1 b_1 = q b,$$
$$a_2 = a,$$
$$b_2 = b;$$

[4] Physically this condition exists when polychromatic light is allowed to fall upon the surface—each color individually is reflected without regard to the presence of the others.

or, in tabular form:

$$\begin{array}{lll} a=r, & a_1=(q/q_1)r, & a_2=r, \\ b=0, & b_1=0, & b_2=0, \\ c=-(1-r^2)^{1/2}, & c_1=\pm(1-q^2r^2/q_1^2)^{1/2}, & c_2=+(1-r^2)^{1/2}. \end{array} \tag{52}$$

If a and c are real, as we are assuming, a_2 and c_2 are also. Hence,

If the incident light is not of hybrid type, the reflected light is not, no matter whether the surface from which reflection occurs is that of a metal or a dielectric. On the other hand, if one medium is a dielectric and the other a metal, one q is real and the other (say q_1) is complex. Hence q/q_1 is not real, whence a_1 and c_1 must be complex, and the refracted light hybrid. The sole exception is normal incidence, for which $r=0$. Hence

Plane-polarized light refracted through the boundary between a dielectric and a metal becomes hybrid, except at normal incidence.

As hybrid light travels more slowly than plane-polarized light, this raises some doubt as to the possibility of determining the refractive index of metals by direct measurement. These will be considered in detail in §20.

This, however, is not the only way of producing hybrid light; for it is possible for q^2r^2/q_1^2 to exceed unity even though both q and q_1 are real, provided $q>q_1$ (that is, provided the first medium is optically denser than the other). Then, though both media are dielectrics, c_1 will be complex, and the refracted beam will be hybrid. We shall find that in such cases there is no propagation of energy into the second medium, wherefore the incident wave is said to be "totally reflected."

There is an essential difference in the character of these two cases, obviously, and we should expect this difference to manifest itself somehow in the nature of the triplet a_1, b_1, c_1. It does so in the fact that, when a *metal* is involved (so that q/q_1 is complex) both a_1 and c_1 are complex; while when both media are dielectrics, a_1 is real and c_1 pure imaginary. This we may indicate by writing

$$\begin{aligned} q_1a_1&=qa+i0, \\ q_1b_1&=0+i0, \\ q_1c_1&=R+iI, \end{aligned}$$

in the one case, and

$$\begin{aligned} q_1a_1&=qa+i0, \\ q_1b_1&=0+i0, \\ q_1c_1&=0+iI, \end{aligned}$$

in the other. It takes only a cursory inspection of these sets to show that in the second case the damping vector is in the z-direction and the velocity vector in the x-direction, as in Fig. 9 (a), while in the first case the velocity is inclined to the system of coordinate axes, as in Fig. 9 (b).

Now we have shown in the first paper that energy always flows in the direction of the velocity vector.[5] Hence in case (a) there is no flow from the first medium to the second.[6] The energy surges and resurges across the boundary, but gets nowhere in the end.

In the other case, part, at least, of the energy which crosses the boundary never returns, but is conducted away. Until we have evaluated the amplitude factors E_1 and H_1 we cannot be assured that this

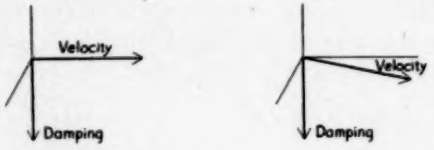

FIG. 9

really means a transport of energy; for if either happened to come out zero there would still be no loss. As we shall see, this latter state of affairs would occur in the case of a perfect conductor, for which E_1 is zero. It, too, is a perfect mirror, not because all the energy which passes it is returned, but because none ever passes.

So much for the triplets a, b, c. Next we wish to evaluate the other triplets and the amplitudes E_1 and H_1. In doing this it is convenient to confine our attention to two cases, one characterized by the fact that E is normal to the plane of incidence, the other by the fact that H is normal to that plane. So long as the incident beam is not hybrid all possible cases can be built up by the superposition of waves of this sort. The typical cases are obviously characterized by the triplets

[5] Except in one case. In that case, however, the Poynting vector has the same component as the velocity *normal* to the bounding surface: the two differ only in their *tangential* components. Hence our present argument is not affected.

[6] The question is sometimes raised: If no net energy flows across the surface, how can there be any energy in the lower medium? The answer is, that the statement only applies to the steady-state condition. When light is first caused to shine on the surface, there is for a time a net flow into the metal; when the light is shut off, a net flow out of it. These, however, are transient effects. During the intermediate period, with equilibrium established, no net flow takes place.

$$\left.\begin{aligned} l&=0\\ m&=1\\ n&=0 \end{aligned}\right\}(E \text{ normal to plane of incidence}),$$

$$\left.\begin{aligned} l'&=0\\ m'&=1\\ n'&=0 \end{aligned}\right\}(H \text{ normal to plane of incidence}),$$

and must be separately considered.

The algebraic process is very simple: We start with one of these triplets, together with the known triplets (52), substitute them in the fundamental equations (3) to (8) of the previous paper, and then apply the three boundary conditions built up in §14. As only the simplest sort of algebra is involved the solution requires only time and patience, and need not be reproduced. The results, however, are important. Listed in tabular form they are:[7]

TABLE 1. *The equations of reflection and refraction.*

E normal to the plane of incidence

$l=0$	$a=r$	$l'=-c$
$m=1$	$b=0$	$m'=0$
$n=0$	$c=-(1-r^2)^{1/2}$	$n'=a$
$l_1=0$	$a_1=qr/q_1$	$l_1'=-c_1$
$m_1=1$	$b_1=0$	$m_1'=0$
$n_1=0$	$c_1=\pm(1-q^2r^2/q_1^2)^{1/2}$	$n_1'=a_1$
$l_2=0$	$a_2=r$	$l_2'=-c_2$
$m_2=1$	$b_2=0$	$m_2'=0$
$n_2=0$	$c_2=\pm(1-r^2)^{1/2}$	$n_2'=a$
$\dfrac{E_1}{E}=\dfrac{k-k_2}{k_1-k_2}$		$\dfrac{E_2}{E}=-\dfrac{k_1-k}{k_1-k_2}$

Notes:

1. The symbol "k" is used for the combination iqc/k, the q, c and k all being understood to have the same subscripts as the k.

2. It is not necessary to write equations for the H's; they are sufficiently defined by means of (9).

3. The ambiguous signs are to be so chosen that the real part of c_1 is negative and the real part of c_2 positive. The significance of other combinations is discussed in the following paragraphs.

[7] I need hardly say that the ordinary laws of reflection and refraction are all contained in this table—not implicitly but *explicitly*. If they are not immediately obvious, it is only because the notation is unfamiliar.

TABLE 2. *The equations of reflection and refraction.*

H normal to the plane of incidence

$l=c$	$a=r$	$l'=0$
$m=0$	$b=0$	$m'=1$
$n=-a$	$c=-(1-r^2)^{1/2}$	$n'=0$
$l_1=c_1$	$a_1=qr/q_1$	$l_1'=0$
$m_1=0$	$b_1=0$	$m_1'=1$
$n_1=-a_1$	$c_1=\pm(1-q^2r^2/q_1^2)^{1/2}$	$n_1'=0$
$l_2=c_2$	$a_2=r$	$l_2'=0$
$m_2=0$	$b_2=0$	$m_2'=1$
$n_2=-a_2$	$c_2=\pm(1-r^2)^{1/2}$	$n_2'=0$
$\frac{H_1}{H}=\frac{\kappa-\kappa_2}{\kappa_1-\kappa_2}$		$\frac{H_2}{H}=-\frac{\kappa_1-\kappa}{\kappa_1-\kappa_2}$

Notes:

1. Here κ is used for iqc/g.
2. The E's are understood to be defined by (9).
3. The real part of c_1 is negative; the real part of c_2 positive.

Every quantity used in discussing our waves is completely determined in these tables, except that the signs of c_1 and c_2 are ambiguous. There is no *mathematical* way of deciding which signs to use, any combination being a solution of the problem. We must therefore revert to the physical meanings of the quantities for their further definition. This can best be done by thinking, for a moment, of a case in which a_1 and c_1, as well as a_2 and c_2, are real. They then represent direction cosines of the velocity of propagation; and in particular c represents the

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For example, the usual law of sines is contained in the equation

$$a_1 = \frac{q}{q_1} r,$$

or

$$\frac{a_1}{r} = \frac{q}{q_1}:$$

for *when the media are dielectrics* the velocities in the two media are $v=p/q$ and $v_1=p/q_1$, as was said in § 4. Moreover, in this case a_1 is the sine of the angle of refraction, and r the sine of the angle of incidence. Hence $a_1/r=q/q_1$ becomes at once $\sin R/\sin I=v_1/v$.

Similarly the amplitude relations, when rewritten in trigonometric notation, are just the usual equations.

The polarizing angle is that angle for which $H_2=0$. From Table 2 it is easily seen to be given by $\kappa=\kappa_1$, which easily reduces to $\cos I/\cos R=v_1/v$, and this, together with the law of sines leads at once to the more familiar relation $\tan I=v_1/v$.

Other common properties appear with equal ease.

vertical component of velocity. Then when c is positive the wave moves upward, and when c is negative it moves downward. Keeping this idea in mind, the four possible cases are seen to be represented schematically by the diagrams of Fig. 10. They may be characterized in words as follows: (a) Two waves, 180° out of phase, superposed. They are equally intense, and therefore interfere destructively. The refracted beam is of zero intensity. The entire case is trivial, but physically possible. (b) The usual case of reflection. (c) Identical with (a), except for the direction of the refracted beam. As this is of zero intensity its direction is obviously inconsequential. Trivial, but physically possible. (d) If a beam were caused to impinge upon the bounding surface *from beneath*, it too would have "reflected" and "refracted" associates. By properly adjusting the angle of incidence, they can be caused to coincide with the beams refracted and reflected from above, respectively.

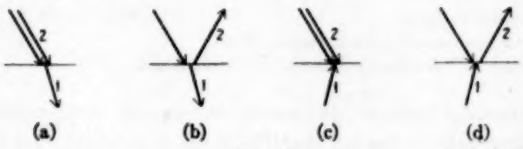

FIG. 10

By properly adjusting the amplitude and phase of the beam incident from below, its reflected beam can be caused to destroy the beam refracted from above. This is the condition of (d). It is physically trivial, but we shall find it possible to build a useful mathematical artifice upon it later on.

If c_1 or c_2 is complex the same set of possibilities is met, except that for "c positive" we must read "the real part of c positive"; and so on.

§16. *Two Numerical Illustrations*: We conclude this part of our study with two numerical illustrations, one of non-metallic reflection, the other of metallic reflection.

For the non-metallic case, we choose glass with an index of refraction 1.5 as our upper medium, and air as the lower one. Then the history of a light wave, as its angle varies from 0° to 90° (in the glass) is told by Fig. 11. The electric vector is supposed to be normal to the plane of incidence.

The upper third of the figure is devoted to direction of propagation, each set of three vectors representing corresponding velocities of incident, reflected and refracted light. As we pass across the figure from left to right the angle of incidence increases, passing from normal

incidence through the angle of total refraction (polarizing angle) and the angle of total reflection to grazing incidence. Until the angle of total reflection is reached, the angle of refraction increases more rapidly

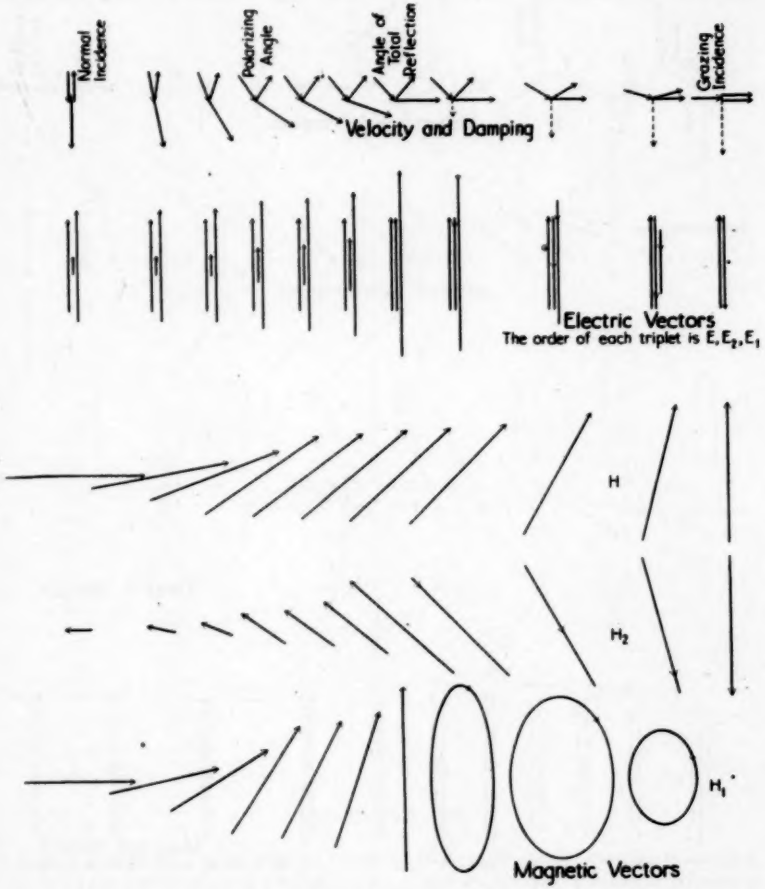

FIG. 11. *Reflection at a glass-air boundary: electric vector perpendicular to the plane of incidence.*

The dotted lines in the upper part of the figure represent the damping vector.

than the angle of incidence and the velocity of the refracted beam remains unaltered; thereafter the angle remains fixed at 90° while the velocity decreases, as shown by the decreasing length of the vector.

At 60° incidence, for example, the refracted beam is moving with only 77% of the usual velocity of light in air. As the velocity decreases, however, a "damping" vector normal to the surface makes its ap-

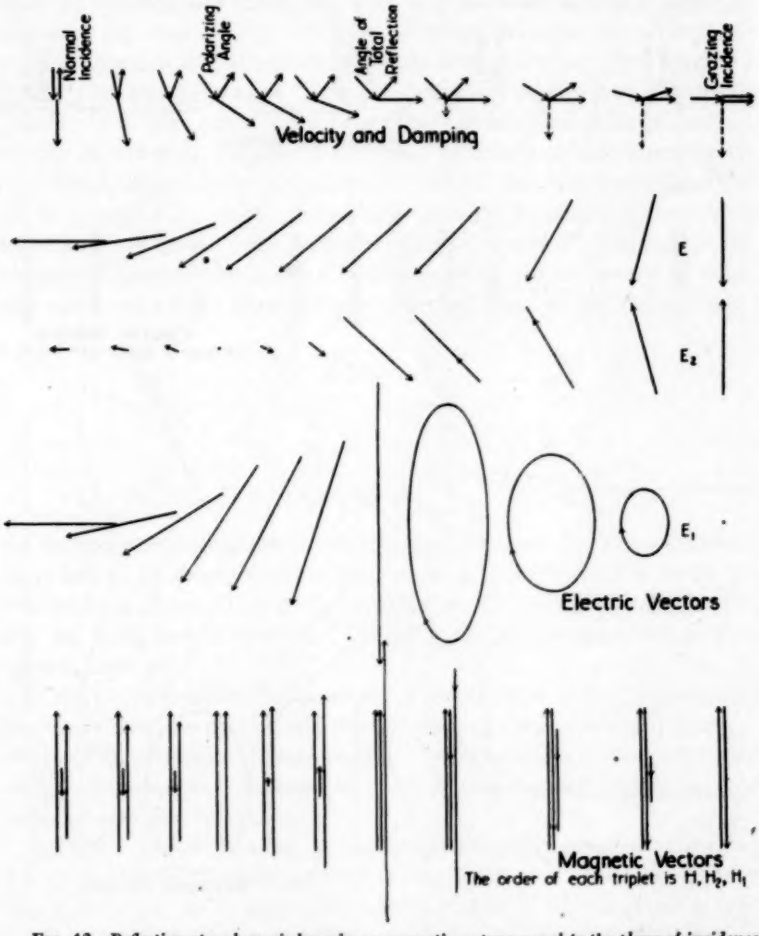

FIG. 12. *Reflection at a glass-air boundary: magnetic vector normal to the plane of incidence The dotted lines in the upper part of the figure represent the damping vector.*

pearance, and rapidly increases in magnitude. It is shown by the dotted lines.

The scale is so chosen that unit length of the velocity vector represents the velocity of light *in vacuo*, while the damping vector gives the "extinction coefficient" directly.

The second section of the figure is devoted to the E-vectors, the top ends of which must be supposed to point away from the reader if the velocity and H are in the plane of the paper. The amplitude of the reflected light increases until the angle of total reflection is reached.

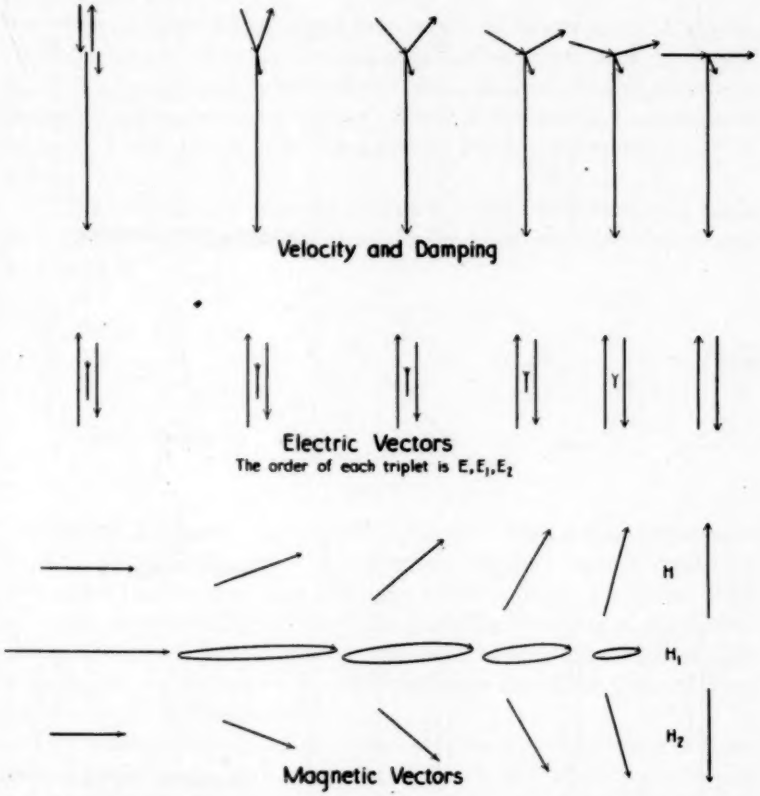

FIG. 13. *Reflection at an air-platinum boundary: electric vector normal to the plane of incidence. In the top part of the figure the vertical vectors, which do not change direction as the angle of incidence changes, represent the damping.*

From this point on it remains constantly equal to the incident vector, but becomes progressively more and more out of phase with it. The refracted vector increases up to the angle of total reflection; then decreases steadily, with constantly changing phase, to zero at grazing incidence.

The remainder of the figure is devoted to the H vectors, which behave in a perfectly familiar fashion until the angle of total reflection is reached. Thereafter, in the case of the refracted light, they describe ellipses in the plane of the paper. As the angle of incidence increases,

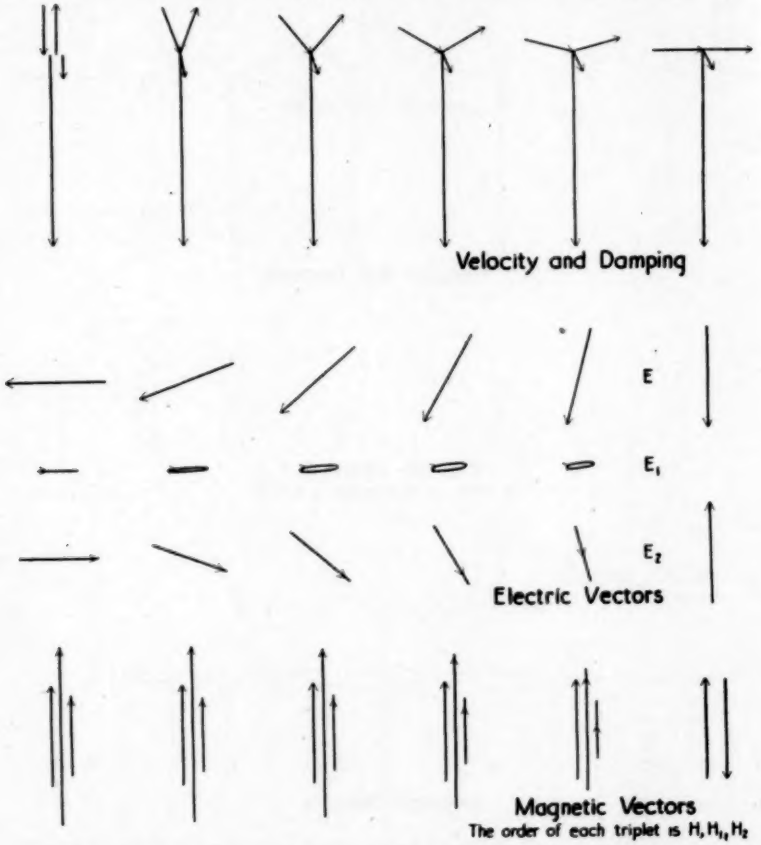

FIG. 14. *Reflection at an air-platinum boundary: magnetic vector normal to the plane of incidence.*

In the top part of the figure the vertical vectors, which do not change direction as the angle of incidence changes, represent the damping.

these ellipses become shorter and fatter and vanish into a point at grazing incidence.

Fig. 12 is similar to Fig. 11, except that H instead of E is supposed normal to the plane of incidence. It needs no comment, except to say

that the tops of the H-graphs must point away from the reader; the others lie in the plane of the paper.

As an illustration of metallic reflection, we may choose platinum for our mirror, and allow the light to impinge upon it from above. The life history is given graphically in Figs. 13 and 14. The principal points to be noted are:[8]

1. The velocity and damping vectors are no longer at right angles. (The damping vectors are the long ones, which point vertically downward. The short ones, which change direction with changing angle of incidence, are the velocity vectors.) There is therefore a component of damping in the direction of propagation, which implies dissipation of energy.

2. The velocity and damping change but little with increasing angle of incidence. In the case of non-metallic total reflection the change was marked.

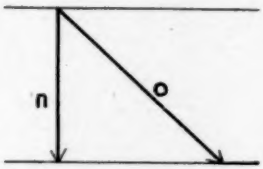

FIG. 15

This has a peculiar consequence, which I suspect has never been noticed. It is, that light which impinges *obliquely* upon a conductor will travel further in it than will light which impinges normally. This is readily seen from Fig. 15; for if the damping is normal to the surface and substantially independent of the angle of incidence, oblique light must travel the distance o to suffer the same attenuation that normal light suffers in a distance n.

This is not in contradiction to the fact observed in the case of semi-transparent liquids, such as dye-stuffs, that the attenuation is proportional to the distance travelled, whatever the angle of incidence; for the conductivity of such substances is low, and the theory then predicts a damping approximately proportional to the cosine of the refraction angle. For substances of *high* conductivity, however, the light penetrates to substantially the same *depth* (not *distance*) regardless of the angle of incidence.[9]

[8] The figures are drawn for a wave length 4346A, the optical constants being taken as $N=2.10$, $K_0=3.80$.

[9] It need hardly be said that the *intensity* at a given depth is not independent of the angle of incidence; for that intensity depends upon *how much light enters* the conductor, as well as

3. Perhaps the most amazing observation is with regard to the behavior of the rotating vectors. In the case of Fig. 13, for example, it is entirely natural for the vector H_1 to be horizontal when light is incident normally; but as the angle of incidence increases and the in-

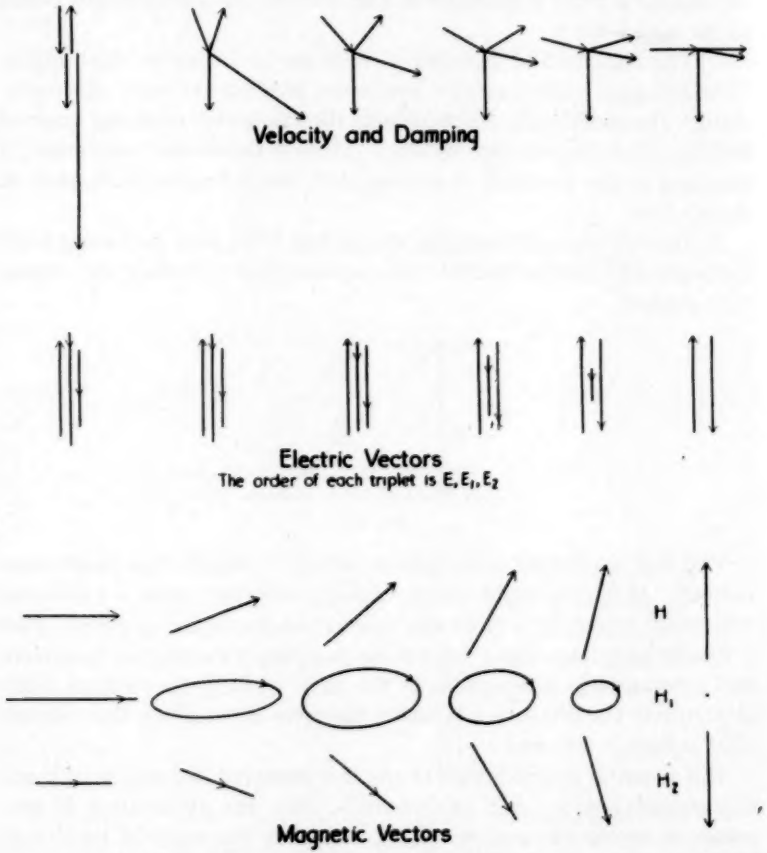

FIG. 16. *Reflection at an air-rubidium boundary: electric vector normal to the plane of incidence.*

In the top part of the figure the vertical vectors, which do not change direction as the angle of incidence changes, represent the damping.

cident H-vector cuts the surface at a greater and greater angle, we should expect the refracted H_1 to tilt upward to a marked degree.

upon *how rapidly it is absorbed.* The former quantity depends markedly upon the angle of incidence; the latter only to a very small extent.

Such a tilt is apparent in the figure, of course, but not to the extent that one would expect. For 75° incidence, for example, the refracted beam is travelling at an angle of 25° from the vertical, yet the long dimension

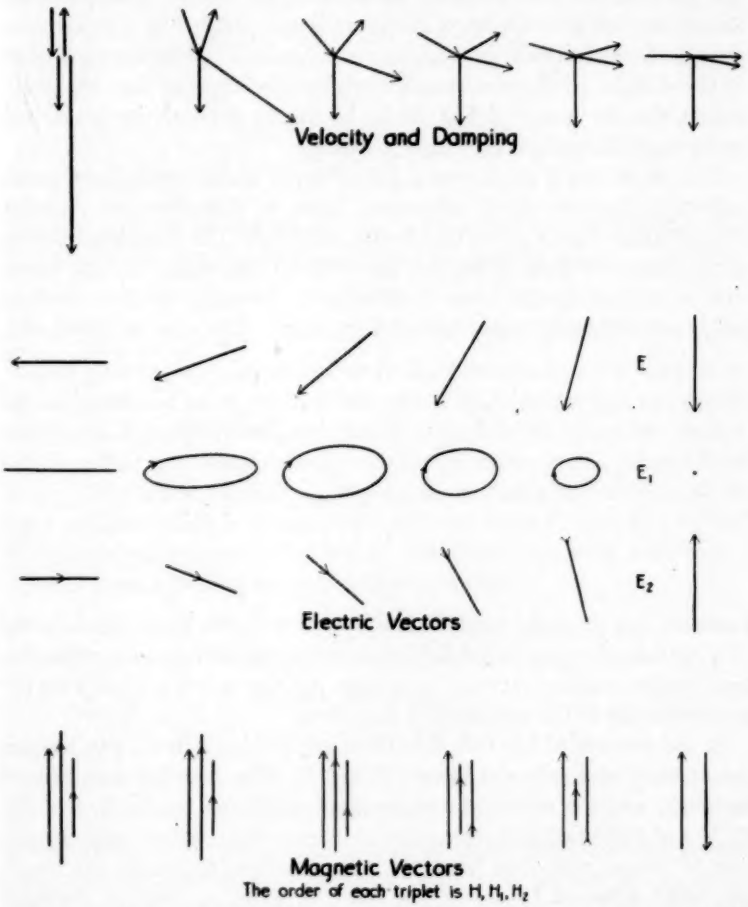

FIG. 17. *Reflection at an air-rubidium surface: magnetic vector normal to the plane of incidence In the top part of the figure the vertical vectors, which do not change direction as the angle of. incidence changes, represent the damping.*

of the H_1-ellipse departs only 6° from the horizontal. It appears to be a property of metals to keep the H and E vectors as nearly as possible parallel to the surface no matter what the angle of incidence may be.

4. Finally, the statements made above are equally as true for metals of low refractive index as for platinum, the refractive index of which is about 2. Thus Figs. 16 and 17, which are drawn for rubidium, exhibit the peculiarities mentioned to an even higher degree than platinum, though the refractive index of rubidium is only about 1/4. For instance, though the direction of propagation in rubidium approaches parallelism to the surface much more closely than for platinum, so that one would expect the H_1-vector of Fig. 16 to be nearly vertical, its major axis really hugs the surface very closely indeed.

§17. *Reflection of Light from a Thin Film of Metal.* Aside from simple reflection, the two most interesting cases of reflection are probably the reflection from a prism and from a thin film. The complete solution of the former of these is beyond the scope of this study, for the reason that it involves the problem of diffraction, the exact solution of which raises mathematical difficulties of high order. The case of a thin film,

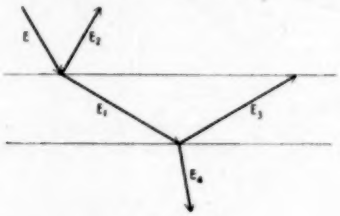

FIG. 18.

however, can be easily treated; moreover it not only illustrates how the idea contained in case (d) of §15 can be utilized in solving such problems, but leads to results that may be of some interest in connection with the determination of the thickness of such films.

In the problem of the thin film there are involved, in all, five beams: the incident and reflected beams E and E_2 (Fig. 18); the transmitted beam E_4, and the refracted and internally reflected beams E_1 and E_3. E_1 is *not* identical with the refracted beam that would exist in the case of simple reflection from the upper surface, for there are many internally reflected beams which coincide with it. But it *could* be produced by simple reflection, if the proper incident intensity were produced, and if there were no internal reflection. Suppose the proper incident intensity were E_5, and the resulting reflected intensity E_6 (Fig. 19).

The internally reflected beam E_3, also, *could* be associated with an incident beam E_7, and a reflected beam E_8 after the manner of Fig. 10

(d), and this combination would have no component in the direction of E_1. Moreover, these two possible arrangements *could* be superposed: if so, the resultant disturbance in the second medium would be identi-

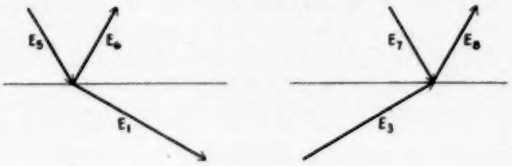

FIG. 19.

cally the same as in the actual case of the film, so the resultant disturbance in the first medium must also be just the same. That is,

$$E = E_5 + E_7,$$
$$E_2 = E_6 + E_8.$$

Finally let us turn our attention to the lower surface. When the resultant refracted beam reaches this point, it no longer has its original amplitude E but has suffered phase, and possibly intensity, change. Let us call its new amplitude E_1', and let us agree that a prime affixed to any other quantity shall, for the moment, designate its value at the lower surface. Then a moment's consideration shows that E_1', E_3' and E_4' are related to one another by the conditions of simple reflection.

From these relations we obtain the equations

$$E_4'/E_1' = 2k_1/(k_4+k_1),$$
$$E_3'/E_1' = -(k_4-k_1)/(k_4+k_1);$$
$$E_1/E_5 = 2k/(k_1+k),$$
$$E_6/E_5 = -(k_1-k)/(k_1+k);$$
$$E_3/E_7 = 2k/(-k_1+k),$$
$$E_8/E_7 = (k_1+k)/(-k_1+k).$$

Finally we must find the relation between the corresponding primed and unprimed quantities. From (2), which is the general law of propagation for our wave, we have

$$Ee^{iq(ax+by)-ipt}$$

as the equation of the disturbance at $z=0$; and

$$Ee^{-iqc\zeta}e^{iq(ax+by)-ipt}$$

as its equation at the surface $z=-\zeta$, ζ being, of course, the thickness of the film. These differ only in that the amplitude factor E has been

replaced by the new amplitude factor $E' = Ee^{-i\omega t}$. Hence we have at once

$$E_1' = E_1 e^{-i q_1 c_1 t},$$
$$E_3' = E_3 e^{i q_1 c_1 t};$$

the positive exponent in the second case being due to the fact that the internally reflected wave moves upward.

By solving these equations we obtain the final results[10]

$$\begin{aligned}
\frac{E_1}{E} &= \frac{2k(k_1+k_4)X^2}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}, \\
\frac{E_2}{E} &= \frac{(k-k_1)(k_1+k_4)X^2+(k+k_1)(k_1-k_4)}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}, \\
\frac{E_3}{E} &= \frac{2k(k_1-k_4)}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)}, \\
\frac{E_4'}{E} &= \frac{4kk_1X}{(k+k_1)(k_1+k_4)X^2+(k-k_1)(k_1-k_4)},
\end{aligned} \tag{53}$$

where

$$X = e^{i q_1 c_1 t}.$$

These equations are written in a form corresponding to the state of polarization in which E is normal to the plane of incidence. To deal with the other case it is only necessary to replace k by κ, and E by H.

When computations are made from these equations they reveal certain peculiarities of reflection from metallic films which are worth noticing in passing.

In the first place, there is a certain thickness which absorbs more energy than does an infinitely thick block of metal, though the film in question is still comparatively transparent. The effect is, of course, an interference phenomenon of the "resonance" type.

In the second place, films which are still so thin as to be virtually transparent exhibit marked selectivity as to plane of polarization, particularly at high angles of incidence, with the result that the state of polarization of the reflected light differs materially from that of the incident light. Since such changes in polarization are easily measured by means of a Babinet compensator, the idea at once presents itself of making them the basis for determining the thickness of such films.

[10] The same results can, of course, be obtained by summing the infinite series of internally reflected components.

With this idea in mind, special attention has been given to the effect of this "selective absorption" upon the reflected light, and some very interesting experiments have been carried out to determine the thickness of "invisible" films of rubidium on glass and platinum. The results of these experiments will be found in a paper by H. E. Ives.[11] Their consistency is probably the best argument in support of the use of a continuous medium theory in dealing with matter which is decidedly "granular." For the purposes of this paper, therefore, we disregard this question entirely, and investigate only the mathematical aspects of the problem.

§18. *The Measurement of the Thickness of Thin Films*: Light polarized at an angle of 45° to the plane of incidence is, effectively, composed of two beams of equal intensity, one with the electric vector *in* the plane of incidence, the other with the electric vector *normal to* that plane. We can, if we like, think of each of these components as producing its own reflected beam: and it is quite obvious that we can easily compute the relative intensities and phases of these reflected components by the use of equations (53). To do so requires some tedious arithmetic but nothing more.

The result we get, of course, will depend upon the thickness of film for which the computations are made, and we could, if we so desired, carry out a set of such computations for various thicknesses.

We could also measure the same quantities by means of a Babinet compensator and its associated Nicol prism, the first of which measures the phase difference directly, and the second the angle θ defined by the equation

$$\tan\theta = \frac{\text{component } E \text{ normal to the plane of incidence}}{\text{component of } E \text{ in the plane of incidence}}.$$

Then, by comparing the *measured* results with the *computed* ones, we could judge the thickness of the film.

Such computations have actually been carried out, in the case of rubidium films deposited on glass, and have led to the curves shown in Figs. 2 and 3 of the paper already referred to.

[11] "The Thickness of Spontaneously Deposited Photoelectrically Active Rubidium Films, Measured Optically," J.O.S.A. & R.S.I., *15*, p. 374; 1927. I take this opportunity to acknowledge my indebtedness to Dr. Ives in connection with this study of Plane Waves of Light. Not only did it have its origin in the attempt to answer certain specific questions which he raised, but it has had throughout the encouragement of his continued interest and the benefit of his knowledge of experimental optics, concerning which I was (and by comparison still am) in a state of profound ignorance.

It was found that the method was a very sensitive one when the optical properties of the film differed materially from those of the backing plate, as was the case with a rubidium film on glass, though with rubidium on a platinum background it did not work so well.

§19. *The Measurement of Optical Constants*: Among the thicknesses of "film" considered in the study to which we have just referred was infinity—which corresponds, of course to bulk material, and in this case the agreement between experiment and computation, as portrayed in Dr. Ives's paper, is excellent. If, however, incorrect optical constants had been used in the computations the agreement would have been poor. In other words, were the optical constants of the metal not known, they could be determined by the requirement that the curves computed from them must agree with the experimentally determined points. This is, in fact, the method usually used for the determination of optical constants of metals, except that it is customary to make use of *one* angle of incidence only, not the whole range; the reason for this course being that the computations are somewhat simpler if it is adopted. Our next purpose is to derive a formula from which the computations may be carried out in general.

To this end we introduce the notation $E_\parallel$ for the component of displacement in the plane of incidence, and $E_\perp$ for the component normal to it. Then the ratio of these two is related to the Babinet reading ϕ and the analyzer reading θ by means of the equation

$$\frac{E_\perp}{E_\parallel} = e^{i\phi} \tan\theta = \rho. \tag{54}$$

This quantity ρ is a complex number, the value of which is known a soon as the experiment has been performed.

Next, we note that the ratio of the normal component of E in the reflected wave to that in the incident wave is given directly by the equations of Table 1. Similarly the ratio of the normal components of H is given directly by those of Table 2; but since both incident and reflected beams are in the same medium, this ratio is the same as that of the components of E which lie in the plane of incidence. From this, and the fact that the two components of E in the incident beam are equal, it follows at once that

$$\rho = \frac{E_\perp}{E_\parallel} = \frac{\kappa_1 - \kappa}{\kappa_1 + \kappa} \frac{k_1 + k}{k_1 - k}.$$

By remembering that $\kappa_1 = iq_1c_1/g_1 = ic_1h_1/q_1$ and $k_1 = iq_1c_1/h_1$, and by assuming that the permeability of the film is unity, we readily throw this into the form

$$\rho = \frac{qc_1 - cq_1}{qc_1 + cq_1} \frac{q_1c_1 + qc}{q_1c_1 - qc} = \frac{qq_1(c_1^2 - c^2) + cc_1(q_1^2 - q^2)}{qq_1(c_1^2 - c^2) - cc_1(q_1^2 - q^2)}.$$

But from (52) we know that

$$c_1^2 - c^2 = r^2(q_1^2 - q^2)/q_1^2,$$

whence we get

$$\rho = \frac{\frac{q}{q_1}r^2 + cc_1}{\frac{q}{q_1}r^2 - cc_1}.$$

Substituting in this equation the value of c_1 as given by (52) and solving for q_1/q we obtain at once the relationship

$$\frac{q_1}{q} = \frac{r}{c(r+1)}[(\rho+1)^2 - 4r^2\rho]^{1/2};$$

or, when we remember that $q_1/q = N + iK_0$, (see §6), and that r and c are the sine and cosine of the angle of incidence,

$$N + iK_0 = \frac{\tan I}{1 + \sin I}[(\rho+1)^2 - 4\rho \sin^2 I]^{1/2}. \qquad (55)$$

No matter what angle of incidence we may wish to use, it is only necessary to substitute the measured values in (54), to obtain ρ, and then substitute this ρ in (55). The result will be a complex quantity of which the real part is the index of refraction and the imaginary part is the extinction coefficient, the latter being defined in the manner described in §6. To obtain the coefficient as defined by Drude and Wood we need only make use of the relationship

$$K = K_0/N.$$

§20. *The Determination of the Optical Constants of a Metal by Direct Measurement*: In §15 we called attention to the fact that, since hybrid light does not travel at the speed of ordinary light, there is some question as to whether the angle through which a beam is deflected by a metal prism is governed by the law that applies to dielectric prisms. To resolve these doubts, we may sketch the following solution of the problem of refraction through a prism.

Suppose light is incident at an angle I to the upper surface of a prism. The direction cosines of the velocity vector of the incident and refracted beams are then, by (52)

$$a = \sin I, \qquad a_1 = \frac{q}{q_1} a,$$
$$b = 0, \qquad b_1 = 0,$$
$$c = -\cos I, \qquad c_1 = -\left(1 - \frac{q^2}{q_1^2} a^2\right)^{1/2}.$$

We denote the angle of refraction by R (See Fig. 20). If the angle of

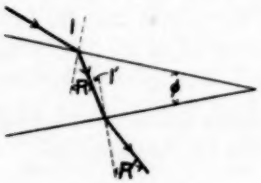

FIG. 20.

the prism is ϕ, the refracted beam meets its lower surface at an angle of incidence $I' = R - \phi$, the direction cosines of which are

$$a' = \sin(R-\phi) = a_1 \cos\phi + c_1 \sin\phi$$
$$b' = 0$$
$$c' = -\cos(R-\phi) = -a_1 \sin\phi + c_1 \cos\phi\ ;$$

and of course the direction cosines of the emergent beam are given at once by (52). They are

$$a_1' = \sin R' = \frac{q_1}{q} a' = a\cos\phi - \left(\frac{q_1^2}{q^2} - a^2\right)^{1/2} \sin\phi.$$
$$c_1' = -\cos R' = -(1 - a_1'^2)^{1/2}. \tag{56}$$

Now the usual practice in determining the index of refraction is so to orient the prism that $I = -R'$: that is, that the angles of incidence and emergence are equal, but on opposite sides of the normal. This means, however, that $a_1' = -a$, whence (56) may be solved for the relation

$$q_1/q = a/\sin\tfrac{1}{2}\phi.$$

If we notice that the total deflection of the ray is

$$\delta = (I - R) + (I' - R')$$

and replace I' by $R-\phi$ and R' by $-I$, we obtain the relation

$$2I=\delta+\phi.$$

Finally, since we are dealing with dielectrics, both q and q_1 are real, whence q_1/q is the index of refraction, N. Thus we have

$$N=\sin\tfrac{1}{2}(\delta+\phi)/\sin\tfrac{1}{2}\phi. \tag{57}$$

This is the usual formula.

Now, turning to the case of a metal prism, we easily find that a_1' is still equal to $a\cos\phi-(q_1^2/q^2-a^2)^{1/2}\sin\phi$ as before, and that c_1' is still $-(1-a_1'^2)^{1/2}$; but these quantities are no longer real, and therefore it is not true that they are the sine and cosine of R'. Instead, this angle is now determined by their *real parts* only. It is possible to write a formula for it, but it is so very complicated that it probably serves no other useful purpose[12] than that of the present investigation, and is scarcely worth reproduction. By finding this angle, however, and setting it equal to $-I$ as before, we can with patience arrive at the result

$$N=\frac{\sin\tfrac{1}{2}(\delta+\phi)}{\sin\tfrac{1}{2}\phi}+(K_0^2-1)\sin\tfrac{1}{2}(\delta+\phi)\sin\tfrac{1}{2}\phi+\frac{N}{N^2+K_0^2}\sin^2(\delta+\phi)+\cdots, \tag{58}$$

the missing terms being of higher order than the second in the sines.

It follows, then, that the formula *does* depend upon K_0, and an index of refraction computed in accordance with (57) would be in error if the prism were of metal, as we suspected in §15. But this is true only in a hypercritical sense; for in practice only very thin prisms, for which both δ and ϕ are small, can be used, and for such prisms the terms which involve K_0 are extremely small. In the case of a platinum prism with a 1° angle, for example, the error introduced by using (57) instead of (58) would be only a tenth of a percent. If it were possible to use a high angle prism, however, the error would be considerable: for a 20° prism it would be about 40 percent.

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JULY 26, 1927.

[12] If we wanted to find the angle of emergence in any particular case we could best do it by computing a_1' and c_1', and using their real parts only in the formula

$$\tan R'=-a_1'/c_1'.$$

THE PHENOMENON OF BROCA AND SULZER IN FOVEAL VISION[1]

By Walter H. Stainton

I. INTRODUCTION

The phenomena associated with the progress of visual sensation as a function of time have interested a large number of investigators. It has long been known that when a bright surface is suddenly exposed to the eye, one does not see the surface at its full brilliancy immediately. It was probably Broca and Sulzer who first observed with stationary fields that there might be a short interval in which a bright field would appear to be much brighter than when viewed for a period of, say, four to six seconds. The quantitative results of Broca and Sulzer[2] are widely known from the curves published in their original notes and reproduced in their later papers[3,4] and in part elsewhere.[5] Somewhat earlier, Bidwell[6] in his experiments on recurrent vision made with rotating disks having luminous sectors, made qualitative observations on this super-brightness effect. Recent work having an important bearing on the present investigation is that of Bills[7] and that of Kleitman and Piéron.[8] Rather extended bibliographies are given in these two papers, and Miss Bills has reviewed much of the work.

The object of the present experiment is to attempt to verify and to extend, particularly to higher brightness levels, some of the work of previous investigators. It will be necessary to describe in a general way the method of determining the visual response as a function of time first used it appears, by Broca and Sulzer and spoken of by them as a "direct method." The original scheme has undergone many changes in its details and it is a modification which is used in the present investigation. Essentially, there are required two luminous

[1] Taken in part from a thesis in the Library of Cornell University.

[2] Broca and Sulzer, C. R., *134*, p. 381; 1902; *137*, p. 944, p. 977, p. 1046; 1903.

[3] Jour. de Physiol. et de Path. gen., *4*, p. 632; 1902; *6*, p. 55; 1904.

[4] Annales d'Oculistique, *131*, p. 107; 1904.

[5] e. g. *Outlines of Applied Optics*, P. G. Nutting, p. 132, Philadelphia; 1912. *Color and its Applications*, M. Luckiesh, p. 138, New York; 1921. *Color Vision*, J. H. Parsons, p. 95, Cambridge; 1924.

[6] Shelford Bidwell, Proc. Roy. Soc., *56*, p. 132; 1894.

[7] Marion A. Bills, Psych. Rev. Monogr., *28*, No. 5; 1920.

[8] Kleitman and Piéron, L'Année Psychologistique, *25*, p. 34; 1924.

fields, preferably juxtaposed, of variable but physically determinable brightness and color, one of which can be exposed for periods of several seconds, the other for periods of perhaps .01 to .5 seconds. Means to make possible accurate fixation must be provided. It must be possible also to compare the two fields as in ordinary equality-of-brightness photometry. With a given objective brightness and energy distribution, one field is exposed for several seconds; then the other is flashed for a fraction of a second. In general it will be possible to judge whether one or the other of the fields is the brighter. The brightness of the long-time or standard field may now be adjusted so as to bring it nearer the apparent luminosity of the flash or test field. The experiment is repeated at intervals until the two fields are judged to be of equal brightness. The brightness of the standard field is now taken as a measure of the apparent luminosity of the test field, and gives one point on the curve of apparent luminosity against time. It will be noted that the experiment might equally well be carried out by fixing the objective brightness of the two fields and then varying the duration of exposure of the test field. The curve obtained by plotting a number of apparent luminosity values against the corresponding durations of flash producing them may be used to obtain a curve of visual sensation against time on the assumption of a function connecting apparent luminosity and visual sensation. For present purposes the question of the relationship can be dismissed by the statement that $S=f(B_{AP})$, where S is the sensation and $f(B_{AP})$ is some function of the apparent luminosity, and that certain conclusions can be drawn knowing B_{AP} while being uncertain of the form of the function.

The results of previous investigations may be summarized very briefly. Broca and Sulzer conclude: (1) The sensations aroused by the fields feebly illuminated take longer times to rise than do the sensations aroused by fields of high brightness. (2) The ratio of the level of maximum sensation to the permanent level increases with increasingly bright fields. (3) With colored fields the maximum of the blue is much higher than for white, much lower for green, and for red, between white and green. (4) The curve after passing the maximum drops down the permanent level in two to four seconds or sometimes longer. They recognized that the exact state of adaptation of the retina could not be known since part was being used for the standard field. It may be noted that they did not assume sensation levels and brightness levels to be identical. This is evidenced by their separate

determination for the observers of the relation between the two on the basis of an extension made by Broca[9] of Fechner's law.

The conclusions of Bills are: (1) The several methods employed (including those of earlier workers) give fairly concordant data for the time of maximum at the intensities used. (2) Different rates of rise are found for the various spectral regions and for white. (3) With increase of intensity, there is a decrease in the time to maximum response. (4) The order in which the maximum is reached varies, for the color used, at different intensities.

Kleitman and Piéron find: (1) The rate of rise is very much greater in peripheral vision than in foveal vision. The result is not entirely unambiguous, as their brightness levels are all expressed in multiples of the threshold brightness as determined for each condition. (2) The rate of change of the "speed of establishment" as a function of the logarithm of field brightness varies with the spectral region but is constant for a given region. The speed of establishment if defined as the reciprocal of the time to crossing the permanent level and is used in preference to the maximum since at low levels the maximum is not well defined. (3) The order of the rates from the greatest, as taken from the slopes of their curves is: red, green, blue, white.

From the summary of results given, it can be seen that the several observers agree that the rate of rise of apparent luminosity as a function of the duration of the exposure of the stimulus increases with increasing brightness level. And here the concordance ends. The high maximum of Broca and Sulzer for the blue is perhaps explainable as Kleitman and Piéron have done on the basis of a super-position of rod and cone effects. The times of maximum obtained by Bills for the three low brightness levels and by various methods are fairly consistent, but are very much shorter than for corresponding levels from the data of Broca and Sulzer and of Kleitman and Piéron. In fact, they are close to the values of time to crossing the permanent or equilibrium level of the others. In Fig. 1 are plotted corresponding data of Kleitman and Piéron and of Bills. In *b* the reciprocal of the time to crossing the equilibrium level is plotted as ordinate with the logarithm of equilibrium brightness level as abscissa, for the several spectral regions used. On the original curve, which is not reproduced, the results for the two have been averaged and straight lines drawn in order from the steepest: red, green, blue, and white. While the separate data for yellow for the

[9] André Broca, Jour. de Physique, III, *3*, p. 206; 1894.

two observers is not given by the authors, it may be mentioned that the curve for the averaged experimental values for yellow has a greater slope than for red. It is important to note that the conclusion of Kleitman and Piéron, namely that the behavior of the eye is different for different spectral regions, would be unchanged by a change in the relative nominal brightness values. Data from Bills most nearly corresponding to that of Kleitman and Piéron is plotted in Fig. 1*a*. Appropriate straight lines drawn would be in order for the colors from the steepest: blue, white, green, yellow, red. Broca and Sulzer and

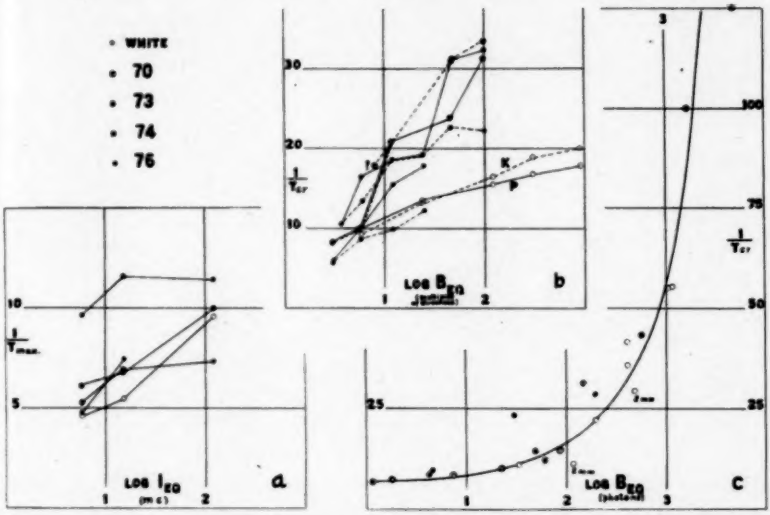

FIG. 1. *(a) Data from Bills. (b) Data from Kleitman and Piéron. (c) Data from present experiment. Key refers to filters of (c), but same symbols are used for corresponding spectral regions of (a), (b) and (c). Points marked "2 mm" refer to data obtained with artificial pupil.*

earlier experimenters have given various orders for the rate of rise, but these probably are not of enough importance to mention. However, from the two sets of data shown, there is sufficient evidence of disagreement.

The experiments mentioned give some measure of the apparent luminosity as a function of the time of exposure of the stimulus and cannot give with certainty the sensation as a function of time. Without further (and very important) assumptions, it is not possible to say that the course of visual sensation as a function of time is being determined.

But without making assumptions, curves of apparent luminosity (or its logarithm) as a function of the time of exposure of stimulus may be obtained and certain conclusions drawn therefrom. Such curves then will exhibit the overshooting of the eye, or as here specifically termed, the phenomenon of Broca and Sulzer.

II. APPARATUS AND PROCEDURE

The apparatus arranged for the experimental work of the present paper is shown schematically in Fig. 2. The essential parts are two light sources, lamps S_1 and S_2; a lens Ls_1 which focuses the light from S_1 in the plane of the rotating sector disk D; a supplementary shutter at M; a reflecting and diffusing screen Sc_1; Lummer-Brodhun cube C;

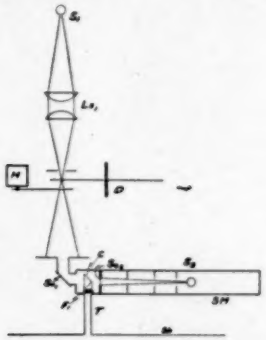

FIG. 2. *Arrangement of apparatus.*

filters at F_1; and sight tube T. The size of the elliptical field of the cube and the length of the sight tube are such that the outside ring or standard field subtends at the eye, a vertical angle of about 2.5°, while the center or test field subtends an angle of approximately 1.5°. A fixation point was provided but found to be unnecessary after the observer had had considerable experience with the ring field. Source S_1 is a 400 watt flat filament projection lamp, rated at 115 volts and burned at 85 volts. The color temperature of this source is approximately 2750°K.[10] S_2 is a 21 cp head-light lamp color-matched to S_1. The motor driven disk D has a single variable sector opening. The shutter M is a semaphore arm with circular opening fixed to the shaft of a direct current motor. An arrangement of stops limits the angle through which

[10] For the determination of the color temperature of a similar lamp burned at the given voltage, the writer is indebted to Dr. Forsythe and Mr. Watson of the Nela Research Laboratory.

the armature of the motor may turn. For indicating the disk speed and hence the durations of flash, the emf of a Weston Magneto Tachometer driven by the disk is read, the necessary calibration being made directly by means of time-piece and a revolution counter on the disk shaft. A check on this calibration is made by a photographic method.

To obtain colored fields, several of the Wratten series of monochromats are used.[11] The total transmission factors here used are for the observer and were obtained under conditions approximating those of the experiment. The wave lengths of the maximum transmission are as follows: #70, 700mμ; #73, 570mμ; #74, 530mμ; #76, 440mμ.

The following, in part, are the instructions to the observer: "It is assumed that the observer will be able to match photometrically the concentric field of the apparatus, the outside (ring) field being viewed for approximately six seconds and the inside field for a flash of short duration. The observer may vary the *brightness* of the ring field by means of a knob under his right hand. The *instant* at which the inside field is flashed is determined by the observer, the key for this purpose being at his left hand. The procedure during a run is as follows:

The observer first makes two settings as in ordinary equality-of-brightness photometry. Then the observer fixates the center of the

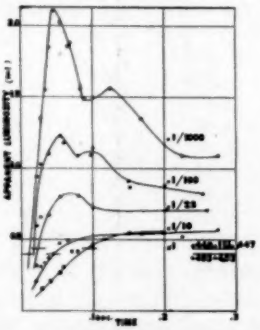

FIG. 3. *Curves for white light.*

(dark) field and by holding the key down causes the center field to flash at intervals of a few seconds, 'bracketing' the apparent luminosity of the center field by adjusting the brightness of the ring field. The bracket from 'certainly too dark' to 'certainly too bright' will be given by perhaps a quarter turn of the knob. The observer then rests for

[11] The total visual transmissions and the transmissions at various wave lengths of the filter films are given in the phamplet *Wratten Light Filters* of the Eastman Kodak Company.

fifteen or twenty seconds, (or until after-images have disappeared) then after six seconds fixation, flashes the center field once. A slight adjustment of the ring field may now be necessary. The periods of rest, followed by single observations are continued until the observer is satisfied with the balance. At the end of a run of a number of points as here described, two more ordinary photometric settings are made. It is of greatest importance that the center of the field be fixated."

All of the apparatus except that mentioned in these instructions is under the control of the operator and also the operator notes all data. The settings made at the beginning and end of a run are to establish the base line or equilibrium level.

III. RESULTS AND CONCLUSIONS

The principal results are given in the form of curves, all for the right eye of the author, (Figs. 3, 4, 5) with specimen data in Tables 1 and 2. The photometer readings in Table 1 are graphical averages

TABLE 1

11/15/16. Dark adaptation. Tachometer calibration: 3. 10 A. M. Obs: W. H. S. Opr: J. E. L. Filter: 73, factor: .035 ml factor: 31 Pupil dia. for B_{EQ} = 1.50 ml is 4.8 mm. Photon factor: 57.5

Mean phot. reading	B_{AP} (ml)	B_{AP} (photons)	Duration (sec.)
.500	.542		.021
.760	.825		.032
1.08	1.17		.045
1.46	1.58		.055
1.28	1.39		.065
1.70	1.84		.115
1.82	1.97		.170
1.90	2.06	118	.225
1.80	1.95		.275
1.38	1.50	86.5	(EQ)

of four individual settings and represent the method used throughout the experiment. In Table 2 are given data in which photometer readings are averaged arithmetically and the average variation from the mean obtained. The process of graphical averaging gives values which are within two or three percent of arithmetical averages; hence the latter were computed only for the data of this table. The two runs given represent the most favorable and most unfavorable conditions respectively.

While not required for the principal conclusions drawn, a correction for variations of pupil area has been applied, making use of the excellent

TABLE 2

11/24/26. Dark adaptation. Tachometer calibration: 3. 10 A. M. Obs: W. H. S. Opr: J.E.L. Filters: 74, 5% neutral, factors: .02, .05 ml factor: 31 Pupil dia. for B_{EQ} = .0410 is 6.4 mm. Photon factor: 102

Mean phot. reading	Av. dev.	B_{AP} (ml)	B_{AP} (photons)	Duration (sec.)
.574	.040	.0178		.038
.645	.035	.0200		.045
.844	.054	.0261		.055
.867	.058	.0268		.065
1.03	.034	.0319		.077
1.14	.023	.0355		.088
1.33	.055	.0412		.115
1.42	.085	.0440		.170
1.55	.058	.0480		.225
1.58	.073	.0490	5.00	.275
1.32	.033	.0410	4.18	(EQ)

10/25/26. 10 A.M. No filters. ml factor: 31. Pupil dia. for B_{EQ} = 45 ml is 3.2 mm. Photon factor: 25.6.

1.94	.12	60.0		.021
3.32	.31	103		.027
3.56	.19	110		.039
3.99	.15	123	3150	.054
3.82	.69	118		.059
3.54	.06	110		.079
3.54	.20	110		.097
3.71	.71	115		.100
2.95	.16	91.5		.150
2.86	.14	88.4		.200
2.68	.23	82.8		.255
1.45	.022	45.0	1150	(EQ)

data of Reeves.[12] The size of the average pupil at fixed brightnesses is nearly inversely proportional to the logarithm of the field brightness for the range here used. It has been assumed that the pupil will be (after exposure of six seconds to the standard field) the size corresponding to the brightness of the standard field. While the absolute size for the individual observer will perhaps differ from the average as given by Reeves, the proportionality quite certainly holds within the limits of error of this experiment. In two instances a check run was made with an artificial pupil, the significant points of the curves agreeing reasonably well with those made without the pupil.

[12] Prentice Reeves, Psych. Rev., *25*, p. 330, 1918; also J.O.S.A., *4*, p. 35, 1920.

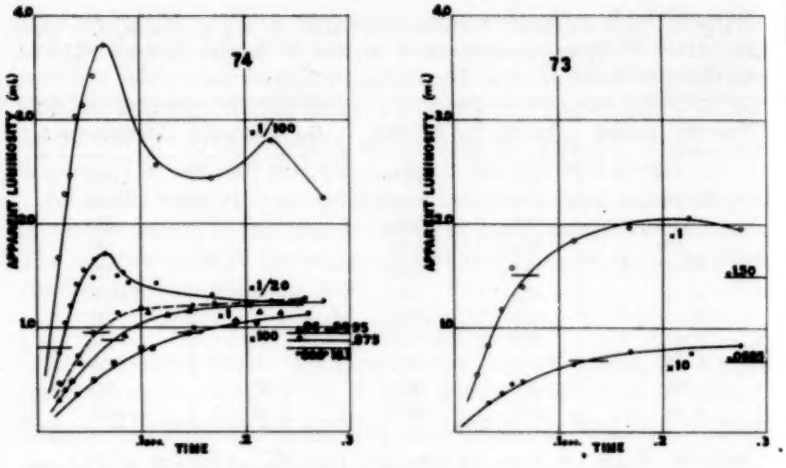

FIG. 4. *Curves for green (filter 74). Curves for yellow-green (filter 73).*

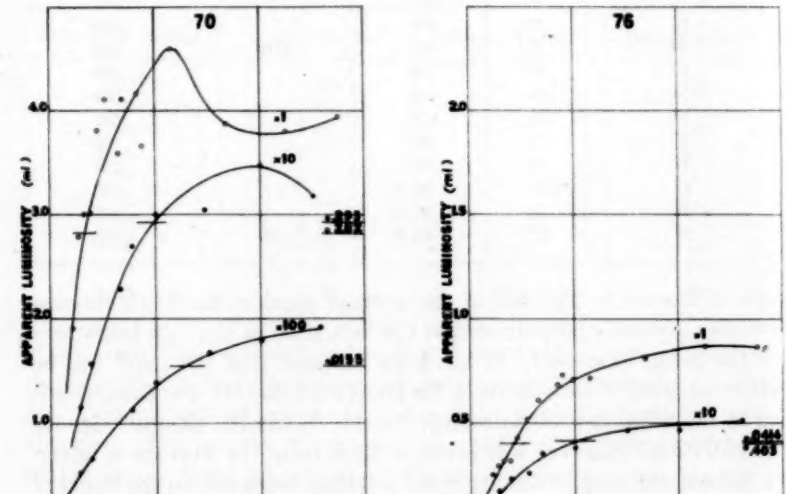

FIG. 5. *Curves for red (filter 70). Curves for blue (filter 76).*

The curves are of course drawn arbitrarily, particularly in that several indicate the possibility of a secondary overshooting effect which while of doubtful existence (at least for the fovea alone) has appeared in the curves of several experimenters.[13] The absolute values of brightness are estimated to be good to ten percent, while the times are probably good to five percent. It will be observed that the ordinates of each curve have been reduced by the factor given on the curve. The important values to be noted are the time to crossing the permanent or equilibrium level, the maximum apparent luminosity, and the ratio of the maximum level to permanent level. The times to the crossing and the maximum, approach zero time at higher and higher levels of brightness and also the ratio of maximum to equilibrium value increases. These relations are exhibited in the curves of Figs. 1*c* and 6. The

TABLE 3

Light adaptation surrounding field: .6 ml tapering to .1 ml

Filter	Approximate brightness of fields compared (ml)	Duration (seconds)	
		Dark adapt.	Light adapt.
none	45	.018	.020
	10	.023	.020
	4.0	.045	.032
70	.30	.10	.090
73	1.5	.070	.058
74	.90	.074	.054
	.010	.15	.076
76	.040	.10	.054

dotted curve (Fig. 4) using filter 74 (green) at .96 ml indicates the resulting increase in the slope when the eye is maintained at a low level of light adaptation, the surrounding field including practically the whole field of view. The time to crossing the equilibrium level under various conditions is given in Table 3. It will be noted that the rate of rise is greater with the daylight adaptation than with the dark surrounding field. For a given level of brightness of surrounding field, the difference between light and dark conditions becomes less with increasing brightness of the fields compared, until at the highest level given (45 ml) the time is the same to within the uncertainty of observation. In this comparison the effect of possible differences of pupil

[13] e. g. Max Büchner, Psychol. Stud., *2*, p. 1; 1907.

size has not been taken into account, hence the comparison is of interest as approaching the conditions of the everyday use of the eye.

Table 4 gives significant data from the curves of Figs. 3, 4, and 5 and a few others which are not shown. Returning now to Fig. 1,

TABLE 4

B_{EQ} (ml)	B_{EQ} (photons)	B_{AP} (max.) (photons)	Log B_{EQ}	$\frac{B_{AP}\text{ (max.)}}{B_{EQ}}$	Duration to crossing (sec)	$\frac{1}{T_{cr}}$
White						
403	4880	25600	3.688	5.25	.008	125
44.0	1150	3150	3.060	2.74	.018	55.5
11.1	401	734	2.603	1.83	.023	43.5
4.03	195	276	2.290	1.44	.045	22.2
.447	33.1	41.4	1.520	1.25	.092	10.9
39.7	1010	2820	3.004	2.79	.018	55.5
39.7	473	970	2.675	2.05	.034	29.4
10.1	404	756	2.606	1.87	.028	35.7
10.1	120	148	2.079	1.23	.095	10.5
70						
2.82	149	244	2.173	1.64	.032	31.2
.293	23.0	27.2	1.362	1.18	.098	10.2
.0155	1.80	2.22	.255	1.23	.128	7.8
73						
1.50	86.5	118	1.973	1.37	.068	14.7
.0685	6.78	8.32	.831	1.23	.130	7.7
74						
80.6	1630	7550	3.212	4.64	.010	100
16.1	563	1210	2.750	2.15	.023	43.5
.875	54.8	77.5	1.739	1.41	.074	13.5
.940	59.0	62.5	1.771	1.06	.082	12.2
.0410	4.18	5.00	.622	1.19	.115	8.70
.00948	1.16	1.39	.064	1.20	.148	6.76
76						
4.03	195	356	2.290	1.83	.035	28.6
.403	29.4	63.3	1.468	2.15	.044	22.7
.0414	4.22	5.25	.625	1.24	.104	9.62

where in part (c) data obtained in the present experiment and given in Table 4 is compared with that of Bills and of Kleitman and Piéron, it will be seen that the results strongly indicate that the eye behaves

similarly for all regions of the spectrum and for white light. The rate of change of $1/T_{cr}$ appears to vary with the equilibrium brightness and to be independent of the spectral region, both of the conclusions being opposed to the directly expressed conclusions of Kleitman and Piéron. Broca and Sulzer also believed that they had shown that the rise is significantly different for various spectral regions. An approximate expression for the curve of Fig. 1*c* is:

$$T_{cr}=\left[\left(\frac{B_{EQ}}{1.47}\right)^{.6}+6\right]^{-1}$$

From this data it appears that T_{cr} approaches a value of about .17 seconds as B_{EQ} becomes very small and as B_{EQ} is made large, the time

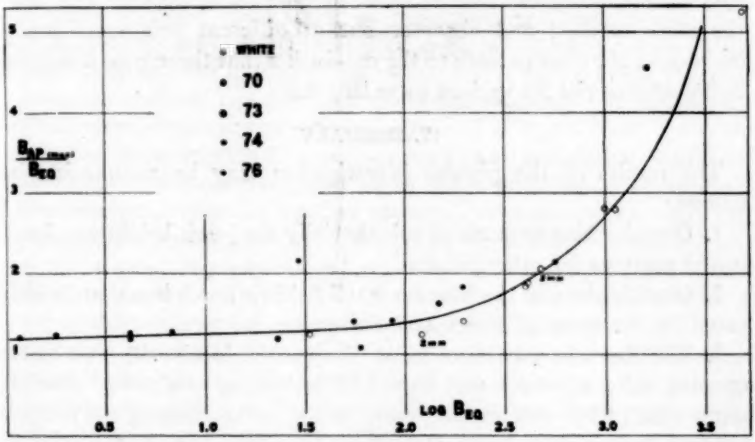

FIG. 6. *Overshooting as a function of field brightness. Key refers to filters. Points marked 2 mm refer to data obtained with artificial pupil.*

to crossing approaches zero. The form of the law is mentioned by Kleitman and Piéron and was probably first suggested by Charpentier.[14]

McDougall[15] in 1904 reported some experiments for "rays of different refrangibilities" which are in accord with the conclusions preceding. The method employed presented the standard and test flashes successively on the same region of the retina. From two series at different brightnesses using red, green and blue gelatine filters, and two other series for red and green, McDougall concluded that "there is no

[14] Aug. Charpentier, C. R. Soc. de Biol., *89*, p. 191; 1887.

[15] W. McDougall, Brit. Jour. of Psych., *1*, p. 151; 1904.

considerable difference between the action-times of red, green and blue light when they are of equal intensities, i.e. of such intensities as to excite sensations that appear to be of equal brightness." The action-time is defined as "the least time during which a light of a given intensity must act on the retina in order to produce the sensation of the highest intensity that it is at all capable of exciting."

Another comparison is presented in Fig. 6, in which the ratio of maximum to equilibrium value of brightness is plotted against the logarithm of equilibrium brightness, and again the evidence is in favor of the view that the behavior of the eye is the same for all wave lengths. While it is perhaps unsafe to say that there are no small differences in the rates of rise for various colors, yet from the wide variety of results by different observers and different methods, and even indeed, with the same method and observer but at different brightness levels, the weight of evidence leads to the conclusion that there is no difference in the rates of rise for various wave lengths.

IV. SUMMARY

The results of the present investigation may be summarized as follows:

1. Overshooting appears at substantially the same brightness levels in the green as for other colors.

2. Overshooting in the blue for small fields is much less than is indicated by the work of Broca and Sulzer.

3. The times to maximum value of apparent luminosity and to the crossing value approach zero time with increasing brightness. For the range used (.01 to 400 ml) and under the given conditions, the relation between crossing time and equilibrium brightness is:

$$T_{cr}=\left[\left(\frac{B_{EQ}}{k_1}\right)^n+k_2\right]^{-1}$$

4. In moderate light adaptation, the rate of rise is greater than for dark adaptation.

5. The ratio of maximum apparent luminosity to permanent or equilibrium brightness increases with increasing brightness, but there is no peculiar characteristic distinguishing various spectral regions.

6. The relation given in (3) does not involve a factor depending upon wave length, hence from this and from (5), it is concluded that the rise of visual sensation is primarily a function of intensity and not of wave length.

The writer wishes to acknowledge his indebtedness to J. E. Lambly and Anne Arnold Stainton for assistance in the laboratory, and to Professor F. K. Richtmyer for many courtesies extended and for his continued interest.

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ITHACA, N. Y.

Pyrometry. By William P. Wood, Associate Professor of Metallurgical Engineering and James M. Cork, Assistant Professor of Physics, University of Michigan. 207 pages. McGraw-Hill, New York. $3.00.

While there are several reference books, now somewhat out-of-date, and many scattered publications on pyrometry, the authors justly feel that such works are not of great utility as an introductory text-book for college students. The present volume gives a brief survey of the practical pyrometric methods together with questions, problems and outlines for laboratory experiments.

Chapter I discusses the Temperature Scale in an entirely too superficial manner even for elementary engineering students. Chapter II on Fluid Thermometers devotes most of the space to gas thermometry. Mercury thermometers in glass and quartz (for service up to 650°C) are mentioned but not a word is presented in regard to their practical applications, immersion, or stem corrections etc. In as much as the student will use a mercury thermometer far more often than a gas thermometer it is quite essential in pyrometry that he be acquainted with the details of such measurements. Several minor criticisms of this character might be made for succeeding chapters on Thermocouples, Resistance Thermometers, Optical and Radiation Pyrometers. The book concludes with three chapters on Temperature Recorders and Controlling devices, Transition points and Thermal Analysis, Refractory Materials Used in Pyrometry, and an appendix showing various pyrometric applications in the industries, temperature conversion tables, melting points, boiling points and thermocouple data.

Outlines for 19 excellent experiments, and 34 practical problems are presented. The volume contains 93 illustrations. The book probably represents the best single treatise available for the class room or laboratory and on the whole should make a highly satisfactory half-year introductory course for third-year students.

PAUL D. FOOTE

A SIMPLE DERIVATION OF THE HYDROGEN ENERGY LEVELS IN WAVE MECHANICS

By Arthur Edward Ruark

For many purposes it is of interest to have a wave-dynamical derivation of the hydrogenic energy levels corresponding to the circular orbits of the Bohr theory. This note presents such a derivation, first neglecting the variability of mass with velocity and then taking account of this variation. The eigenfunctions are not correct since they depend only on the azimuth ϕ, but the energy values are the usual ones. The virial theorem shows that

$$-\overline{T}=\overline{V}/2=E$$

where T, V, and E are kinetic, potential, and total energy respectively, and bars indicate time averages. But T and V are constant for a circular orbit, of radius r. Then,

$$T=p_\phi^2/2mr^2=-E \tag{2}$$

and

$$-Ze^2/2r=E \tag{3}$$

Eliminating r from (2),

$$p_\phi^2+Z^2me^4/2E=0 \tag{4}$$

We form the wave equation by replacing p_ϕ by $h/2\pi i\,\partial/\partial\phi$ and applying to ψ the operator thus obtained from the left side of (4). It is,

$$d^2\psi/d\phi^2+k^2\psi=0 \tag{5}$$

where

$$k^2=2\pi^2me^4Z^2/-Eh^2$$

The solution is $e^{ik\phi}$, and to make this single valued, finite, and continuous as a function of ϕ, E must take the values given by the classic theory of hydrogen energy levels.

In relativity mechanics, the virial theorem is

$$-\overline{T'}=\overline{V}/2 \tag{6}$$

where $T'=mv^2/2(1-\beta^2)^{1/2}$, m being the rest mass of an electron and v its velocity, so that $v=\beta c$. T' and V are constant, so $-2T'=V$. We substitute this value of V in the energy equation

$$mc^2\left(\frac{1}{(1-\beta^2)^{1/2}}-1\right)+V=E$$

obtaining

$$1-\beta^2=\left(1+\frac{E}{mc^2}\right)^2\equiv A. \tag{7}$$

The angular momentum is

$$p=\frac{m\beta c}{(1-\beta^2)^{1/2}} \tag{8}$$

and from (6)

$$\frac{m\beta^2c^2}{(1-\beta^2)^{1/2}}=\frac{Ze^2}{r} \tag{9}$$

By the aid of (7) and (9) we eliminate r and β from (8), obtaining

$$p_\phi^2-\frac{Z^2e^4}{c^2(1-A)}=0,$$

an equation analogous to (4). Proceeding exactly as before, the wave equation is (5), where

$$k^2=\frac{4\pi^2Z^2e^4}{h^2c^2(1-A)}=\frac{\alpha^2Z^2}{1-A},$$

α being the fine structure constant of Sommerfeld; k must be an integer, and we find that

$$1+\frac{E}{mc^2}=\left(1-\frac{\alpha^2Z^2}{k^2}\right)^{1/2}$$

in agreement with the usual formula for the energy of circular orbits in relativity theory. The Zeeman effect and the modifications due to the spinning electron can be treated in similar fashion.

The selection principle for the azimuthal quantum number is easily obtained by considering the integrals

$$M_x=\int\Psi\bar{\Psi}\cos\phi\,d\phi, \qquad M_y=\int\Psi\bar{\Psi}\sin\phi\,d\phi$$

In determining the radiation from the atom M_x and M_y are supposed proportional to the x and y components of electric moment of the atom, and

$$\Psi \equiv \sum c_k e^{i(k\phi - 2\pi\nu t)}$$

The arbitrary coefficients c_k are supposed real, and $\bar{\Psi}$ is the complex conjugate of Ψ. Substituting this value of Ψ in the expressions for M_x+iM_y, M_x-iM_y, we have to deal with the typical integrals

$$\int_0^{2\pi} e^{i(k-k'+1)\phi}d\phi, \qquad \int_0^{2\pi} e^{i(k-k'-1)\phi}d\phi.$$

The first is zero unless

$$k-k'+1=0\,;$$

The second is zero unless

$$k-k'-1=0.$$

It is interesting to note that ψ_k should be taken as $e^{ik\phi}$, not as $\cos k\phi$ or $\sin k\phi$; for if

$$\Psi=\psi_k e^{-2\pi i\nu t}$$

we may take the real part of Ψ as the wave function, namely,

$$\cos k\phi \cos 2\pi\nu t+\sin k\phi \sin 2\pi\nu t$$

This represents a *rotating* amplitude distribution, while the functions $\cos k\phi \cos 2\pi\nu t$ and $\sin k\phi \sin 2\pi\nu t$ represent a pulsating phenomenon when considered separately.

2. It is usually stated that the normal Zeeman triplet is predicted by the use of the relativistic wave equation for the hydrogen atom in a uniform constant magnetic field. I find that if terms of the order E/mc^2 are retained, (E being the sum of kinetic and potential energies), the solution of the wave equation can be interpreted as follows:

An observer in a reference frame rotating about the lines of force would see the Ψ-distribution approximately as though no magnetic field were present provided his angular speed is

$$\frac{eH}{4\pi mc}\left(1+\frac{E}{mc^2}\right)$$

Since E is negative for "elliptic orbits" this speed is always less than that of the Larmor precession as determined by classical mechanics.

When $E \to 0$, it approaches the speed of the Larmor precession. Inclusion of the term E/mc^2 means that the variability of electronic mass with velocity is taken into account. This result is identical with one obtained by Pauli[1] on the old mechanics, using the relativistic variation of mass, except that in his equation E is the energy measured in the *rotating* system, while our E is measured in a resting frame. The difference is only an apparent one, for both derivations neglect terms in H^2.

GULF PRODUCTION COMPANY'S FELLOWSHIP,
MELLON INSTITUTE OF INDUSTRIAL RESEARCH,
UNIVERSITY OF PITTSBURGH,
PITTSBURGH, PA.
SEPTEMBER, 1927.

[1] ZS. f. Physik, *31*, p. 373; 1925.

Polarization of resonance-radiation. The experiments are performed with the Hg vapor at room-temperature or 0°C. (pressure of 10^{-3} or $2 \cdot 10^{-4}$ mm) excited with plane or elliptically polarized light; the direction of observation is inclined at 160° to the primary beam. Plane-polarized exciting light produces resonance radiation which in the absence of magnetic field is plane-polarized to a high percentage; as a magnetic field applied parallel to the direction of observation is increased past 0.5 gauss, it is seen that the percentage of polarized light is decreasing and at the same time the plane of vibration of the polarized part is turning in the "proper" sense, i.e. as if the "vibrators" in the Hg atoms were undergoing Larmor precession. An estimate of the half-period of the postulated damped vibrators leased to 10^{-7} sec. Admixture of hydrogen cuts down greatly the intensity of the resonance radiation but not its degree of polarization; admixture of argon has the reverse effects. Separate experiments are performed to prove that the rotation of plane of polarization is not a Faraday effect. Elliptically polarized exciting light produces resonance radiation which (viewed at 160° as before) is elliptically polarized in the same way, if there is no magnetic field; when the field is applied as before the resonance rays are a mixture of circularly-polarized and unpolarized light, while those excited by circularly-polarized light remain circularly polarized—a result from which, by combination with the known facts for plane polarized exciting light, the observations with elliptic light may be predicted. [W. Hanle (Göttingen), ZS. f. Phys. *30*, pp. 93–105; 1927.]

KARL K. DARROW

PHYSICAL OPTICS

REPORT OF PROGRESS COMMITTEE* FOR 1926–27

A. Standard Wave Lengths
B. Spectrum Structures
C. Measurements of Short Time Intervals
D. Standards of Optical Planeness
E. Velocity of Light
F. Absorption and Magnetic Rotation at Low Temperature
G. Polarized Light from Incandescent Filaments.
H. Photoelectric Effect
I. Dispersion
J. Compton Effect
K Refraction and Total Reflection of X-rays
L. Absorption of X-rays
M. Measurement of X-ray Wave Lengths
N. Polarization of X-rays
O. Electron Reflection.
P. Efficiency of an X-ray Tube
Q. Resume of X-ray work from 1921–1925.

A. *Standard Wave Lengths.* A contribution to the measurement of secondary standards in the arc spectrum of iron was made by Kleinewefers[1] who compared the wave lengths of 51 lines (5167.493–6677.994) with that of the red radiation from a quartz-cadmium arc. The result of all the investigations of this nature during the past decade is to cast doubt on the suitability of an iron arc operated at atmospheric pressure as a source of standards, and considerable evidence that the vacuum arc is more satisfactory has accumulated.

The "Preliminary Table of Solar Spectrum Wave Lengths," published by Rowland more than 30 years ago, has been of inestimable value in the development of spectroscopy and astrophysics. The need for its extension and correction to the new international scale has been felt for many years. It now appears that this revision is well advanced. Beginning with the 201 standard solar wave lengths (4073.767–4754.047A) compared with neon standards by Burns and Meggers[2] in 1925, this program has been extended during the past year (1) by Babcock's[3] study of the infrared solar spectrum with the interferometer in which 507 lines (6868.915–8980.519A) are expressed on the neon scale, and (2) by Burns and Kiess[4] who have directly determined the wave lengths of 350 lines (5805.226–7148.159A) in terms of neon standards. The accurate determination of wave lengths corresponding to several thousand Fraunhofer lines throughout the spectrum will ultimately make it possible to interpolate the remaining lines and thus put the entire list of solar spectrum lines on the international scale.

* The members of this committee are R. T. Birge, L. R. Ingersoll, W. F. Meggers, F. L. Mohler, H. M. Reese, J. H. Van Vleck, and W. P. Davey, *Chairman.*

B. *Spectrum Structures*: The analysis of spectral structures goes on apace although there is a noticeable slump in the classification of complex spectra, probably due to the fact that all available descriptions of Zeeman effects had been exhausted a year or more ago. Since that time, additional data have been obtained on the Zeeman-effect of tin[5] and tungsten[6] lines; similar observations in many other spectra are sorely needed. It may be pointed out that no data for the magnetic resolution of spectral lines have ever been published for any of the 14 rare earth elements nor for the following: As, Hg, P, S, Se, Ta, U, and Xe.

The theory of spectral terms and electron configurations outlined and referred to in the last two reports has been fully developed in a book by Hund[7] and many applications of this theory to the interpretation of spectra have been made. Thus Fowler and Freeman[8] have discussed the structure of the N_{II} spectrum, and Russell and Meggers[9] have found the structures of the Sc_I and Sc_{II} spectra in complete agreement with the theory.

With the interpretation of the Balmer series of an alkali-type doublet spectrum by Sommerfeld and Unsöld[10] and the resolution of the so-called "doublet" lines of He as triplets by Houston[11] and by Hansen[12] two anomalies have been removed, and the spectroscopic alternation law (that even values of the multiplicity r occur for odd values of the atomic number Z, and odd values of r for even Z) now has no apparent exceptions.

Another phase of spectroscopic analysis, the investigation of the hyperfine structures of spectral lines, is attracting more attention; during the past few months a preliminary statement on the hyperfine levels in the La_I and La_{II} spectra has been published by Meggers and Burns[13] and a remarkable analysis of the fine structure components of Bi_I lines has been made by Goudsmit and Back[14]. These investigations suggest that the moment of momentum of the atomic nucleus is responsible for the phenomena, but the data themselves are probably still too meager for a satisfactory development of the theory.

C. *Measurement of Short Time Intervals*. J. W. Beams, with various collaborators, has made effective application of a modification of the method of Abraham and Lemoine for measuring short time intervals, to a number of interesting optical problems. A Kerr cell between crossed nicols is connected with a spark gap by leads of variable length so that the relaxation of the field (i.e., closing of the "shutter"), may be delayed by a definite amount.

The method was first applied to determining the order of appearance of various spectral lines.[15] It now has been refined and improved so that the actual time interval between the appearance of various lines has been measured for the spectra of magnesium, cadmium, zinc, hydrogen, nitrogen,[16] mercury, and helium.[17] The results emphasize the importance of the final as well as the initial energy levels in determining the time between excitation and emission. For the last two elements the time intervals are found to be independent of conditions of excitation.

The method has also been modified to allow the obtaining of short definite flashes of light,[18] and has been applied to a preliminary study of the zinc spark. Using two liquid-filled cells in solenoids, the same general scheme has also been used to study[19] the differences in time lags of the Faraday effect behind the magnetic field, in various liquids. With the exception of hydrochloric acid, all the liquids show a somewhat larger lag than carbon bisulphide, which is taken as the comparison liquid, the difference being from 1 to 4×10^{-9} sec. There is no simple relation with the Verdet constants or magnetic susceptibilities.

D. *Standards of Optical Planeness.* An achievement in optical workmanship, at the Bureau of Standards, of more than passing interest is described by Skinner.[20] Three fused quartz discs about 25 cm diam. by 4 cm thick, were ground, polished and tested in pairs by means of interference fringes, until the surface of each showed a divergence from a true plane of less than 1/100 of a wave length. If these discs prove to be permanent in form, as may reasonably be expected, they will furnish self-checking standards of planeness of extraordinary accuracy and practical freedom from temperature effects.

E. *Velocity of Light.* Michelson[21] has continued his measurements on the velocity of light, over the 22 mile base between Mt. Wilson and Mt. San Antonio, Calif. He has now used in turn five different revolving mirrors of glass or steel, with from eight to sixteen facets each. The results for the velocity *in vacuo*, given by the separate mirrors, show a remarkable agreement as follows:

Glass	8 facets	299,797
Steel	8 "	299,795
Glass	12 "	299,796
Steel	12 "	299,796
Glass	16 "	299,796

The mean result is 299,796 ± 4 kms/sec. The experiments are shortly to be continued with the far mirror on Mt. San Jacinto, 82 miles distant.

In an appendix, Wm. Bowie of the Coast and Geodetic Survey describes the precautions taken in measuring the base line. The probable error is one part in 6,800,000, which is considered a greater accuracy than has ever been attained hitherto in triangulation measurements.

F. *Absorption and Magnetic Rotation at Low Temperatures.* Becquerel, Onnes and de Hass[22] report the results of a study of absorption and magnetic rotation in certain rare earth compounds (xenotime, tysonite, parisite and bastnaesite) at very low temperatures, continued from 1908. In general the absorption spectra of these crystals at liquid air temperatures become simpler when the temperature is lowered to the freezing point of hydrogen (14°*K*), and, on the whole, still simpler at the boiling point of helium (4.2°*K*). However, while some lines disappear at these very low temperatures, new ones also come in. The effect of the magnetic field on these absorption bands has also been studied and found to be quite complicated. Dissymmetries appear in the intensities of the components corresponding to oppositely circularly polarized waves, and, down to 140°*K*, these dissymmetries result, as a rule, in an increase in intensity of the components displaced towards the shorter wave lengths.

The results on magnetic rotation of the plane of polarization show that this continues to be approximately inversely proportional to the absolute temperature down to 4°*K*. The rotation, however, increases slightly more slowly than $1/T$ from room temperature down to 20°*K*, and still more slowly from there down to 4°K. The rotation dispersion curves given for the temperatures 4.21°K and 20.36°K show a very rough general proportionality of the rotation to the inverse square of the wave length. The authors feel that there are two essentially different kinds of magnetic rotation, one due to the (inverse) Zeeman effect and the other to the dissymmetry in the absorption of opposite circular waves, brought about by the field.

G. *Polarized Light from Incandescent Filaments.* Worthing[23] has investigated the polarization of the light emitted by incandescent tungsten, tantalum and molybdenum, and the deviations from Lambert's law. The results for the three metals are similar and show a polarization increasing from zero at normal emergence up to about 95% at grazing emergence. This means, as a whole, the light emitted normal to the axis by a straight-filament vacuum tungsten lamp may be some 15% polarized.

The Lambert's law deviations are in such a sense as to show a brightness increase of about 20% at emergence 75°, over that at normal. The brightness then decreases rapidly towards zero at grazing emergence.

The change in optical constants of tungsten with temperature have also been studied. A small but definite effect is observed, which checks well with the observed values of emissivity for normal emergence.

H. *The Photoelectric Effect.* Several papers have appeared dealing with the effect of impurities upon the threshold wave length, or upon the saturated photoelectric current. H. K. Dunn[24] has followed the method of a flowing mercury surface, used by Kazda in fixing the threshold of that metal. (See report of this committee for 1926). He verifies Kazda's value of 2735A for the threshold of pure mercury, and shows that changes in the threshold value and the sensitivity occur very quickly if the technique is altered in any way that will admit impurities, although traces of hydrogen delay these changes. His results are reproducible.

L. A. Du Bridge[25] has studied the effect of prolonged outgassing of platinum. Using foil from three different sources, he finds that in each case the same limiting value for the photoelectric current is reached after several hundred hours of heating at 1200 to 1400 degrees C., at a pressure of 10^{-7} to 10^{-8} mm. This he regards as the true photoelectric current of pure platinum. By the use of absorbing screens in the path of the illumination from the quartz mercury lamp, he locates the threshold at 1958 ± 15A. Illuminating the foil while still hot, and taking the difference between the electronic emissions when illuminated and when not illuminated as the photoelectric emission, he finds that the latter increases above 500 degrees C. He explains this by supposing that the kinetic energy of the electrons, becoming appreciable at such temperatures, is added to the absorbed quantum to help balance the work function term in the quantum relation.

H. E. Ives[26] measured with varying temperature the electronic current from a strip of platinum carrying a thin sublimated film of caesium, with and without illumination from an outside source. The difference, taken as the photoelectric current, is of the same order as the thermionic current, and each shows a maximum about 800 degrees K. He attributed the maximum to progressive attenuation of the caesium film. Referring to O. W. Richardson's suggestion ("The Emission of Electricity from Hot Bodies," 2nd ed.) that thermionic emission may be basically photoelectric emission provoked by the radiation of the emitting metal, he concludes that this is not the case. Otherwise it should be possible,

at the temperatures in question, to double the total electronic emission by illuminating the feebly glowing strip from an external source of the same intrinsic brightness. This does not occur.

H. E. Ives and G. R. Stilwell[27] have studied the "normal" and the "selective," photoelectric currents from sodium-potassium alloys of varying composition, with undispersed radiation from a tungsten filament. Curves are given for each, and for their ratio, as a function of the atomic percent of sodium. Differences in these curves are attributed to atomic groupings in the surface with definite orientations, responding differently according to the azimuth of polarization. It is stated in a footnote that the current for the "normal" effect fails to show proportionality with the intensity of the incident light, further information being promised for a later article.

W. H. Crew[28] also gets the photoelectric current from heated metals as the difference, under full illumination from a water-cooled mercury arc and no illumination, using platinum and tungsten. The pure metals show no increase due to illumination, but oxide-coated filaments do. Use of absorbing screens indicates that waves longer than 3000A have some effect, but that shorter waves account for most of the current. There are indications that the core of the line at 2536A is especially effective. This matter should be investigated further, since abnormal efficiency within a very narrow wave length range would seem highly improbable. Variation of the applied potential difference brings about a maximum value of the measured photoelectric current instead of saturation, for which fact no explanation is assigned by the author.

N. R. Campbell contributes two papers[29] dealing with the characteristics and the use of potassium cells, sensitized by the Elster and Geitel method, containing argon and a little nitrogen. Several methods are described for the use of such cells, giving very high sensitivity. An important detail is the use of a thermionic valve as a current limiting device, in series with the cell, working on the saturation part of its own characteristic.

F. C. Toy, H. A. Edgerton, and J. O. Vick[30] working in the laboratories of the British Photoelectric Research Association, failed to find any parallelism between the photoelectric and photographic properties of the silver halides.

S. C. Roy[31] finds that, for a number of metals, all that were investigated, the total photoelectric emission varies with the temperature of the illuminating tungsten filament, as thermionic emission from the metal in question would vary with its own temperature. This fact

recalls Richardson's suggestion of a connection between the two effects, and the author concludes, after a plausible theoretical discussion, that the thermionic phenomenon, despite its much greater order of magnitude as ordinarily observed, is probably a case of photoelectric emission stimulated by the radiation in the emitting metal. In this connection, however, reference should be made to the paper of Ives mentioned earlier in this report.[26]

F. Wolf[32] measures the velocity of photoelectrons in the normal and in the selective effect, as stimulated by the same monochromatic frequency. He finds essentially the same velocity in each case, using a sodium-potassium alloy, illuminated by a mercury arc, 365 being the shortest wave length used.

I. *Dispersion, etc.* P. A. M. Dirac[33] presents a quantum theory of dispersion in which the radiation field is treated as a dynamical system, whose interaction with an ordinary atomic system may be described by a Hamiltonian function, acting as a perturbation term. The assumption is made that this term is small, which means physically that the mean life time of the atom in any state is large compared to its periods of vibration. The theory is not strictly consistent with the restricted principle of relativity, though the relativity variation of mass with velocity is taken into account. The resulting dispersion formula is the same as that obtained by Kramers and Heisenberg.[34]

A paper by V. K. Prokofiev[35] deals with the numbers of resonating electrons, according to the classical dispersion theory, involved in the dispersion near the potassium doublets of the principal series. Contrary to Miss Baxter[36] he finds that the ratio of the number associated with the first doublet to that associated with the second, is the same for any vapor density, having the value 110 to 1.

G. C. Darwin and W. H. Watson[37] have examined existing data on the magnetic rotary dispersion of various materials, and found them consistent with Becquerel's formula

$$V = \frac{e\lambda}{2mc^2} \frac{dn}{d\lambda}$$

(V being Verdet's constant), yielding a constant value of e/m of the proper order of magnitude. Oxygen gas is an exception for which no explanation is offered.

J. *Compton Effect.* The existence of the Compton Effect[38] has been amply verified not only by direct measurements of wave lengths by P. A. Ross,[39] S. K. Allison and W. Duane,[40] R. Glocker,[41] G. E. M.

Jauncy,[42] O. K. DeFoe,[43] and Y. H. Woo,[44] but also by investigations on the direction of the scattered x-rays and recoil tracks by Compton and Simon,[45] F. Kirchner,[46] O. Darbyshire,[47] and E. C. Watson.[48] B. Rajewsky[49] claims to have found a multiple scattering, corresponding to a multiple Compton constant, but no confirmation of this seems to to be in the literature. If experiments on the Compton effect are attended under certain experimental conditions by a diffraction of characteristic rays from the crystal[50] such a diffraction effect must be negligibly small.[51, 52, 53]

TABLE 1. *Indices of refraction for x-rays*

Wave Length	Substance	$1-\mu$ exptl.	$1-\mu$ calc.	reference
0.6311A	Pyrites	2.82×10^{-6}	2.62×10^{-6}	58
0.7078	Paraffin	0.701 ± 0.03		59
	Celluloid	0.880 ± 0.03		59
	Carbon	$1.23\ \pm0.15$		59
	Sulphur	$1.39\ \pm0.5$		59
	Aluminium	1.68	1.77	60
	Aluminium	$1.68\ \pm0.07$		59
	Silver	$5.85\ \pm0.03$		59
	Copper	$5.95\ \pm0.5$		59
1.389	Pyrites	13.2	13.5	58
1.389	Copper	35		61
1.537	Paraffin	$3.28\ \pm0.15$		59
	Celluloid	$4.78\ \pm0.1$		59
	Glass (composition not stated)	8		61
	Aluminium	8.4	8.36	60
	Aluminium	$8.40\ \pm0.2$		62
	Mica	9.35		62
	Pyrites	17.6	17.6	58
2.498	Mica	26.4		62
3.378	Mica	49.2		62
3.447	Mica	51.4		62
5.166	Mica	106.5		62
7.109	Mica	186.2		62
8.319	Mica	249		62

K. *Refraction and Total Reflection of X-rays.* A resume of early work is given by A. M. McMahon in the Nov., 1926 Journal of the Franklin Institute.[54] In 1925, M. Siegbahn[55] not only studied the refraction of x-rays in terms of the deviations from Bragg's law of diffraction, but also studied the phenomenon directly with a glass prism, using the K-radiation of Fe, Cu, and Mo. The direct method was also used by M. de Broglie and J. Thibaud[56] who reflected the K-radiation from

Cu, Mo, and W, from "mirrors" of Al, glass, Cu, Ag, Au, Pt, and Pb. W. Linnik and W. Laschkarew[57] reflected Cu radiation from glass, alum, steel, mica, quartz and calcite, and from a silvered surface. In the case of the silvered surface they report surface reflection up to a glancing angle of 2 degrees with an error of less than 2 minutes of arc. A. Larson, M. Siegbahn and I. Waller[63] have not only refracted a beam of x-rays, but have succeeded in separating Fe K alpha, Fe K beta, Cu K alpha, and Cu K beta by dispersion. Table 1 gives a list of indices of refraction in terms of $\delta = 1 - \mu$ for various substances for various wave lengths. Further data are quoted in "X-rays and Electrons" by A. H. Compton, (Van Nostrand).

L. *Absorption of X-rays.* The effect of (1) slit-width, (2) slit-height, and (3) distribution of energy on the focal spot of the x-ray tube, have been carefully studied by F. K. Richtmyer.[64, 65] His measurements on the *K*-absorption discontinuity in Mo, Ag, Sn, Au, and Pb support the formulas

$$\frac{\mu_k}{\rho} = k_k\lambda^3 + \frac{\sigma}{\rho}$$

$$\frac{\mu_l}{\rho} = k_l\lambda^3 + \frac{\sigma}{\rho}.$$

His results are tabulated in Table 2. S. J. M. Allen[66] has applied Wetzel's photoelectric formula to give, in terms of Schrödinger's wave-

TABLE 2. *Mass absorption coefficients for wave lengths on both sides of the K limit.*[65]

Mo	Ag	Sn	W	Au	Pb
375	545	595	1870	2230	2570
50	70	90	330	395	476
7.5	7.8	6.6	5.65	5.65	5.40
13.3	11.0	8.90	3.19	2.57	2.37

mechanics, a theoretical formula for the mass absorption coefficient. His general formula is

$$\frac{\mu_k}{\rho} = k_1\lambda^3 + k_2\lambda^{3.5} + k_3\lambda^{2.5} + \frac{\sigma}{\rho}.$$

Formally, this reduces to Richtmyer's equation if k_2 and k_3 equal zero. Practically his value of k_1 is different enough from Richtmyer's to

give finite values for k_2 and k_3. Thus instead of Richtmyer's 545 for k_1 in the case of silver he uses

$$\frac{\mu_k}{\rho} = 400\lambda^3 - 20\lambda^{3.5} + 80\lambda^{2.5}.$$

E. C. Stoner and L. H. Martin[67] have critically compared the published results of Allen, Hewlett, and of Wingardh with their own and with certain results of Richtmyer. Their tabulations are too extensive for reproduction here. They conclude that their own results and those of Richtmyer are the most accurate.

The coefficients of absorption of celluloid and of air for very soft x-rays have been computed by E. R. Laird[68] in terms of the square of the time of duration of thermoluminescence of a solid solution of 2% $MnSO_4$ in $CaSO_4$. At 40 volts (minimum wave length 310A) the celluloid showed no measurable transmission. At 610 volts (minimum wave-length 20A) the transmission was 57% for a thickness of 25 mμ. For air, μ/ρ is 6.0, 6.25, 7.0, 7.6×10^3 at 600, 500, 400, and 300 volts respectively. Her results are consistent with Richtmyer's cube-law for wave lengths. S. J. M. Allen[69] has measured the mass-absorption coefficients of C, paraffin, S, and sixteen metals between Al and U, for wave lengths from 0.56 to 0.08A. His results fit the empirical formula

$$\frac{\mu}{\rho} = C\lambda^{2.92} \cdot \frac{N^4}{A} + \frac{\sigma}{\rho}$$

C is 0.0132 for the K series and 0.00181 for the L series.

W. B. Morehouse has shown[70] that a combination of Moseley's Law with the absorption law indicates a dependence of the absorption coefficient upon valence. Experimentally, the absorption at the L limits for iodine is about 0.4% less for I^- in NaI than in the element. This agrees with the calculated result.

G. A. Lindsay and G. D. van Dyke[71] have studied the absorption edge of Ca^{++} in calcite, gypsum, and fluorite. They interpret the fine structure of the edge as a demonstration of virtual orbits occupied temporarily by electrons ejected from the K orbit.

M. *Measurement of X-ray Wave Lengths.* (a). In general long x-ray wavelengths have been measured by four methods, (1) in terms of the photoelectric effect, (2) by crystals with large spacings, (3) by ruled gratings, (4) by the calorimeter method.

(1) The photoelectric method measures, by means of a retarding potential, energies of photoelectrons ejected by soft x-rays, and

translates the maximum energies into wave lengths by means of the quantum relations. The results obtained by various experimenters seem to be quite consistent for each worker, but vary somewhat from worker to worker. Descriptions of carefully planned experiments are given by U. Andrews, A. C. Davies and F. Horton,[72] G. K. Rollefson,[73] C. H. Thomas,[74] K. T. Compton and C. H. Thomas,[75] and O. W. Richardson and F. C. Chalkin.[76] Points of disagreement and their probable causes are pointed out by A. C. Davies and F. Horton.[77] Closely allied to this sort of work are the determinations of critical potentials in secondary electron emission by R. L. Petry[78] and H. E. Kreft.[79] The data of all the above are too voluminous to tabulate in this report, and the original publications must be consulted.

(2) Crystals of large interatomic spacings (palmitic acid, $d=35.59$; lauric acid, $d=27.268$; gypsum, etc.) have been used by R. Thoraeus and M. Siegbahn,[80] R. Thoraeus,[81, 82, 83] J. H. van der Tuuk[84] and A. Dauville[85, 86]. Data obtained by this method are free from the objections to method (1) and are quite consistent from one worker to the next. Here again the data are too numerous to tabulate in this report and the original papers must be consulted. The elements studied are B,[86] C,[85, 86] F,[80, 81] Sc,[84] Cr,[80] Mn,[80, 81] Fe,[80, 81, 86] Co,[81] Ni,[81] Cu,[81, 82] Zn,[81, 82] Ga,[84] Ge,[82] As,[81] Br,[81, 82] Rb.[81, 82] The M series of W has been studied by R. V. Zumstein.[87]

(3) A. H. Compton and R. L. Doan[88] have measured the characteristic wave lengths of Cu and Mo, using a ruled speculum grating having 200 lines per mm. J. Thibaud[89] studied the spectra of C, Mg, Al, Fe, Cu, and Pb, between the limits of 140–6000A, using a vacuum spectrograph with a glass grating having 200 lines per mm. F. L. Hunt[90] using a similar grating has studied the K alpha lines of C and Al and the L alpha lines of Cr, Fe, and Cu.

(4) A new method has been introduced into x-ray spectroscopy by by W. R. Ham.[91] The material to be investigated is made the target of a water cooled Coolidge Tube, and the heat generated at the target is compared with the electrical energy input into the tube for various retarding potentials between the anode and the glass walls of the tube. This makes it possible to plot, for any tube voltage, the ratio of the number of incident electrons from the cathode stream to the number of secondary electrons emitted from the target as a function of the velocities of the secondary electrons. This ratio suddenly increases when the tube potential is made to exceed one of the critical potentials of the anode material. In this way a flow-calorimeter is made to act

as a spectrometer in which wavelengths are given from absolute energy measurements by means of the quantum relation.

(b) Considerable work has been done on the measurement of the shorter x-ray wavelengths. The K series of Mo has been redetermined by S. K. Allison and A. H. Armstrong.[92] M. Siegbahn and E. Hjalmar[93] have made precision measurements on Cu K alpha. J. Schror[94] has made precision measurements on the K series for Cr, Mn, Cu, Zn, Se, Yt, Cb, Mo, Ru, and Rh, and on the L series for Sm, Gd, Tb, Dy, Ho, Er, Ad, Cp, Ta, W, Pt, Tl, Pb, Th, and U. A. Leide[95] has used the Siegbahn-Seeman spectrograph to measure in the K series the $alpha_1$ and $alpha_2$ lines for elements from Cu to W, the $beta_1$ and $beta_2$ lines from Cu to I, and the $beta_3$ line for Mo, Pd, Ag, Cd, and Sn. He reports a new line, $beta_1$ for Mo and Pd. B. E. Stephenson and J. M. Cork[96] have measured accurately the K series of all the elements from Ta to Bi except Hg. G. Kellestrom[97] has made similar measurements on the K series of Pd and Ag.

The L radiation from radium B during its spontaneous transformation is very interesting. Before the expulsion of the beta ray, radium B has an electron configuration corresponding to atomic number 82. After the electron is ejected the atomic number is 83. E. Rutherford and W. A. Wooster[98] find that the natural emission spectrum of radium B corresponds to that of element number 83, thus adding to the evidence that the characteristic spectra are caused by electrons *falling into* orbits, not by being *ejected from* orbits.

(c). The so-called *J* phenomenon[99,100,101,102] still refuses to die a natural death in spite of the evidence of direct wave length measurements. It has been observed directly in C, O, Al, Cu, Pt, and Au, and if real is evidently tied up with absorption rather than emission.

(d). Corresponding to the effect of chemical state upon absorption spectra, noted above, E. Backlin[103] finds an effect of chemical state on emission spectra. When the element S is compared with S^{++++++} in the SO_4^{--} ion the following shifts were observed on the photographic plate,—S K $alpha_2$ to SO_4 K $alpha_2$, 0.361 mm; S K $alpha_1$ to SO_4 K $alpha_1$, 0.362 mm; S K $alpha_4$ to SO_4 K $alpha_4$, 0.362 mm; i.e. the shift is uniform and depends only on the chemical state. Similarly the shift from Si K $alpha_2$ to Si K $alpha_4$ was 7.640 mm while for SiO_2 it was 7.823 mm.

(e). Precision measurements have been made on the relative intensities of lines in the K series of Mo and Cu, and on the L series of W by

S. K. Allison and A. H. Armstrong.[92,104] A. Johnson[105] has studied the intensities of the L series of Mo, Rh, Pd, Ag, Cd, In, Sn, and Sb.

N. *Polarization of X-rays.* Ever since the original work of C. G. Barkla in 1906, and the work of W. R. Ham in 1910 on polarization of x-rays there has been an intermittant interest in the subject. Three papers have been published on this subject, during the last three years. A. H. Compton and C. F. Hagenow[106] have shown that heterogeneous x-rays scattered by paper, aluminium, and sulphur are almost completely polarized. The results cannot be accounted for on the basis of fluorescent x-rays from the radiators, as these are almost completely unpolarized. This work was extended by J. B. Bishop[107] who showed that the K alpha lines given off by an Mo-target x-ray tube are at least partially polarized, and by H. Mark and L. Szilard[108] who found the normal polarization effect for the K alpha rays from copper.

O. *Electron Reflection.* Last year reference was made to a research by Dymond on the angular distribution of electron scattering in helium. The work has now been published in full[109] and deserves further mention. He determined for various initial speeds both the velocity distribution of scattered electrons at a fixed angle of 5°, and the angular distribution of electrons which had lost 20 volts kinetic energy. At 5° the velocity distribution curves with V volts initial velocity show a high peak at V volts (elastic collisions), a lesser peak at $V-20.5$ volts (excitation from $1\,{}^1S$ to $2\,{}^1S$). Higher excitation stages give much smaller peaks which merge into a smooth curve with a maximum near $V-50$ volts. Hughes and Jones [110] have made measurements of scattering at 90° and record only elastic collisions at that angle. The results on angular distribution of electrons which have lost 20.5 volts are complicated. A principal maximum which accounts for most of the electrons moves from 24° at $V=50$ to smaller angles which soon become unmeasurable with increasing voltage. There is a second maximum which moves from 50° to 70° as V is increased from 50 to 400 volts; while above 200 volts a third sharp maximum appears at an angle of 30° and does not change with V. As concerns the principal maximum, the result is consistent with a measurement by Langmuir and Jones[111] by a much simpler method. They found for $V=100$ volts that 22 percent of the scattering collisions gave a loss of 20 volts with the most probable angle of scattering at 18°, while 34 per cent of the collisions are elastic with an angle of 23°.

The remainder presumably give ionization with energy losses greater than 25 volts and correspond to the continuous part of the velocity distribution curves of Dymond.

The complicated angular distribution curves resemble results obtained by Davisson and Kunsman[112] for scattering by a metal surface. They studied elastic scattering by ordinary micro crystalline surfaces of several metals. For lower electron speeds the distribution curve was a simple lobe in the direction of the incident beam. For higher speed electrons and the heavier metals other maxima appeared in directions which varied with the electron speed.

Classical physics and the Bohr theory offer no satisfactory explanation of the more complicated curves, but the new wave mechanics suggests in a general way how such patterns may arise.[113] It pictures an electron beam of momentum mv as a wave train of wave length $\lambda = h/mv$ and from this view point scattering of electrons is equivalent to diffraction, and under suitable conditions may give a complicated diffraction pattern .

The outstanding experimental result of the year has come from a continuation by Davisson and Germer[114] of the scattering experiments using a single crystal of nickle. This was mounted with a (111) surface normal to the electron beam and was etched to expose facet surfaces. For a random setting of crystal and voltage the angular distribution curve was commonly a simple lobe such as was observed at the lower voltages with other surfaces. But, with the crystal oriented so that reflection from a crystal plane was in the plane of the receiver, a series of angular distribution curves at small voltage steps showed other lobes. These would appear, reach a maximum and fade with increasing voltage, at the same time moving to smaller angles with the normal. Readings with fixed voltage and receiver angle, as the crystal was turned around its normal, showed that the intensity of these lobes depended upon the crystal symmetry like a reflection from a crystal plane.

Thirteen sets of such lobes were studied in the range from 54 to 174 volts, and the voltage, crystal orientation, and receiver angle for the maxima determined. An analysis of the data led to the following correlation between electron reflection and x-ray diffraction. If the structure of the nickle crystal were contracted by a factor .7 along the direction of the incident beam, then among all the diffracted x-ray beams, the ten of longest wavelength correspond to ten of the electron reflections where the equivalent wave length of the electron beam is given by the wave mechanics relation $\lambda = h/mv$. This relation is derived by the use of the atomic spacing in the crystal surface, and is independent of the contraction factor. Three of the electron reflections were

not accounted for by the single crystal structure, but Patterson[115] points out how certain local imperfections in the crystal may explain these.

The electron speeds used correspond to wavelengths between 1 and 2 angstroms. The physical conditions of electron reflections are in some respects very different from x-ray reflection.[116] For one thing the scattering power of atoms is enormously greater for electrons, so that the surface layer of atoms plays a predominant part and the crystal reflects more like a crossed grating than a space lattice. Hence an appreciable range of equivalent wavelengths can be reflected with a fixed angle of incidence through various reflection angles. The introduction of the contraction factor is equivalent to assuming that the crystal has a refractive index of .7. Because of this the angle of maximum reflection is not equal to the angle of incidence referred to the effective crystal plane.

In conclusion it should be noted that these electron scattering experiments involve a difficult technique; and, because of the several independent variables, the amount of data required is very great. It seems safe to predict that these preliminary results may mark a new era in atomic physics and that future experiments will be concerned more with the momentum exchanges between electrons and matter than with the energy exchange.

P. *Efficiency of an x-ray tube.* H. M. Terrill[117] finds that the total output of x-ray energy from an x-ray tube is 25×10^{-6} of the energy input at 30 Kv and 19.2×10^{-4} at 100 Kv.

Q. *A resume of work from 1921–1925.* A resume of work in x-ray spectroscopy from 1921 to 1925 has been published by A. E. Lindh.[118]

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Polarization of resonance radiation. In Keussler's experiments the resonance-radiation λ2536 of mercury is analyzed by sending it through a nicol into a photoelectric cell, and plotting the photocurrent against the azimuth of the nicol; the resulting curve is of the form $y = A \sin^2 (x-a) + B$, and when the constants A, B, a are determined from the curve the degree of polarization of the light and the plane of its polarized constituents are calculated directly. In the observations the exciting radiation is plane-polarized, the direction of observation is normal to its electric vector and coincides with the direction of whatever magnetic field is applied; the pressure of Hg vapor is $2 \cdot 10^{-5}$ mm. The degree of polarization declines from 80% to 20% as the magnetic field is increased from 0 to 1.3 gauss; the curve is smooth but not exponential (it has a point of inflexion). The electric vector of the polarized constituent is rotated from parallelism with the exciting vector to an inclination of some 35° as the field is increased from 0 to 1.3 gauss. Both curves agree "perfectly" with formulas derived by postulating a classical bound electron subject to radiation forming a continuous spectrum of uniform intensity (the same formulas were developed by G. Breit, This Journal, 10, pp. 439–52; 1925). The half-period (damping constant) of the postulated bound electron is estimated from the data as $1.13 \cdot 10^{-7}$. The discrepancies between these results and those of Wood and Ellett (notably the smaller percentage of polarization and the departure of the curve from the exponential form) are attributed by von Keussler to the superiority of his method. [V. von Keussler, München Technische Hochschule; Phys. ZS., *27*, pp. 313–316; 1926.]

KARL K. DARROW

The origin of the auroral green line. The best determination of the wave length of the auroral green line being that performed by Babcock with a Fabry-Perot interferometer, McLennan and McLeod use the same instrument upon that line which the former and his collaborators produce in oxygen with or without admixture of argon and identify with the auroral line. Babcock announced the value $5577.350 \pm .005$ I.A. for the latter; McLennan and McLeod obtain the value $5577.341 \pm .004$ I.A. for the former. The value given for the widths of the two lines likewise agree. [J. C. McLennan and J. H. McLeod (Toronto); Proc. Roy. Soc., *A115*, pp. 515–527; 1927.]

KARL K. DARROW

INSTRUMENT SECTION

A COMPARATOR-MICROPHOTOMETER

By George R. Harrison

In making intensity measurements in complex spectra the lines are often found so closely packed that they are not sufficiently resolved except with the highest dispersion and a very narrow slit. In certain important regions of the titanium spectrum, for example, many lines which must be separately measured are less than 0.1 A apart. For general spectrophotometric work it is often desirable to use only the first order of a concave grating on account of overlapping of orders and other considerations, which with a 10 meter grating ruled with 15 000 lines per inch, means a dispersion of about 1.67 A per mm. Important lines may thus be separated by less than 0.06 mm.

A common method of comparing line intensities involves the use of a slit sufficiently wide that a single maximum reading may be taken as proportional to the area of the line, the slit image being broader than the natural half-breadth of the widest line. However, even where the lines are generally comparatively narrow, as in titanium, there often exist noticeable differences in line shape when a slit 0.06 mm wide is used. This means that a number of densitometer readings over the breadth of the line must be taken so that its area can be computed, and a microphotometer is required which will have sufficient resolving power to give these readings meaning. Thus it is desirable that a portion of the image not over 0.01 mm wide be measurable, to give a number of readings across a single line. Several excellent microphotometers have been designed[1] which appear to fulfill this requirement, and in addition possess great steadiness and sensitivity. These, however, have generally been designed with automatic recording primarily in mind. It seemed to the writer that improvements in ease and convenience of manual and optical operation might be possible, as there are certain types of photometry where the self-recording feature is of only secondary importance.

In addition it seemed that the usefulness of an ordinary comparator would be greatly increased if the operator could read the intensity of a

[1] Moll, Proc. Phys. Soc. London, *33*, p. 207; 1921.
Pettit and Nicholson, This Journal, *7*, p. 187; 1923.
Siegbahn, Phil Mag., *48*, p. 217; 1924.

line at the same time that he read its wave length. Wave length setting would be greatly facilitated also, since it is often difficult with the eye to determine the exact center of gravity of a diffuse line, while with the microphotometer-comparator, as the galvanometer reading might be made to vary by as much as 1000 scale divisions across a single line, the position of minimum deflection could be easily determined. The present paper describes such an instrument whose behaviour fulfills all expectations. It is arranged to be used as a recording microphotometer also when desired.

Fig. 1 shows a view of the completed instrument in position on the desk on which it is used, but with the galvanometer reading telescope removed. The housing *T* contains one of the improved vacuum thermo-

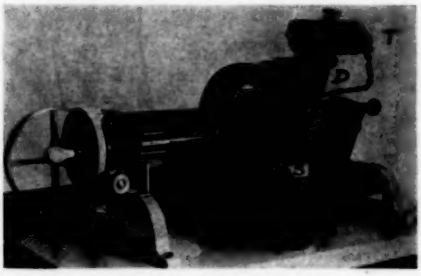

FIG. 1. *Photograph of the comparator-microphotometer ready for use in wave length and intensity determinations, with the galvanometer reading telescope removed. The thermoelement container T can be removed and an ordinary eyepiece substituted for it.*

elements described by Moll and Burger[2]. This was obtained from P. J. Kipp & Zonen. This element marks a very great advance in apparatus of its type and leaves little to be desired in sensitivity and freedom from drift. It is connected to a Leeds & Northrup moving-coil galvanometer, of resistance 17 ohms and sensitivity 6.16 mm per microvolt. A scale at 5 meters is used, raising the sensitivity to over 30 mm per microvolt. The external critical damping resistance is 12 ohms, approximately equal to the resistance of the thermoelement.

In Fig. 2 is given a diagram of the optical system. The lettering used is the same as that in the photograph. A straight filament lamp *L*, drawing 3.0 amps and having a large bulb to reduce blackening, is run from a 12 volt storage battery. The light from this passes up through a diagonal glass plate *P* whose function is to deflect air currents. This

[2] Moll and Burger, Phil. Mag., *50*, p. 618; 1925.

plate was found to greatly increase steadiness of deflection in the densitometer described in a previous paper,[3] which also had a vertical optical system. The light is focused on the slit S by a 32 mm microscope objective O_1, the image being one-fourth natural size. The slit S is 3 mm by 0.008 mm, and is practically in contact with the emulsion side of the photographic plate to be measured, which is held on the carriage C.

In most microphotometers an image of a lamp filament is cast on a slit, and a reduced image of this slit is thrown by a microscope objective on the emulsion to be measured. Unfortunately the chromatic aberration resulting from the incomplete correction of the lens for all wave

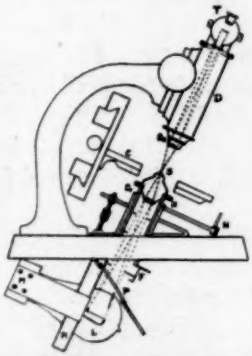

FIG. 2. *Diagram of the optical system of the comparator-microphotometer.*

lengths between about 3.0 μ and 0.35 μ reduces the resolving power somewhat, and the extra lens decreases compactness and brightness. To avoid the second difficulty the slit and first lens are sometimes omitted, with a resulting sacrifice of resolving power but a considerable increase in brightness. In the present instrument (as in the densitometer previously described[3]) one lens was omitted, with a resulting increase in deflection obtainable, and the slit was placed practically in contact with the emulsion, increasing the compactness and resolving power greatly.

It was found that deflections of 1000 mm on an easily read scale were obtainable with the slit only 8 μ wide, although wider slits may be inserted in the holder H. The slits are made by screwing two jaws on the sides of a tube bevelled at 45°, the bevelled sides of the jaws

[3] Harrison, This Journal, *10*, p. 157; 1925.

being placed upward, so that they are parallel. The tube holding slit and lens O_1 can be moved up and down by a rack and pinion, operated by turning a handle N which projects from the front of the instrument. When a new plate is placed on the carriage C the slit is advanced to within a few microns of the emulsion, and a stop is then tightened which prevents further advance. The spreading of the light beam before it strikes the emulsion slightly increases the effective slit width, but scattering in the emulsion and lack of complete parallelism between slit and spectrum line increase it by a greater amount, and are present in any type of microphotometer.

The light, after passing through the plate, is focused by a second 32 mm microscope objective O_2, on the filament of the thermo-element. This filament can be viewed from above through a small lens, and the magnified image of the element can be set very accurately over the beam of light. The element is 100 μ wide, and as the slit image is magnified four times, it should cover but one-third of the width of the element. Chromatic aberration of the lens 0_2 in the infrared, however, blurs this image somewhat, and it is focussed for maximum deflection of the galvanometer.

The head T containing the thermoelement can be readily pulled from the focusing tube with its draw-tube, and an ordinary 5x Hyperplane eyepiece, fitted with cross-hair, pushed in its place. This gives 20-fold magnification of a spectrum line, and when the slit has been pulled from its mounting, and a ground glass screen pushed into place at the bottom of the lower tube, the instrument is converted into an ordinary comparator. It was originally hoped that the lines might be watched visually and the galvanometer readings observed at the same time, but this introduced complications which it was not thought worth while to attempt to overcome at present. It would be of great advantage to be able to watch the plate while taking measurements, to discriminate between specks and lines, for while it is true that the microphotometer will show up real effects which the eye cannot see, as has often been emphasized, it will also report as real, effects which the eye can see to be false.

No detailed description need be given of the comparator proper, as its construction is quite orthodox, except that in making the castings due attention was given to the requirements of a microphotometer. The plate carriage C is 15 by 24 cm, and has a six inch traverse. It is driven by a 1 inch screw having 1 mm pitch, and the divided head and vernier enable readings to be taken to 0.001 mm. Provision is made

for a slow-motion drive, a small hand wheel turning a worm which can be made to engage a wheel fixed to the screw shaft. In addition a notched wheel was placed on the worm shaft; this is provided with a phosphor-bronze spring which clicks once for each advance of the plate of 0.0005 mm. A 6-inch pulley wheel is mounted at the rear of the worm shaft, so that the screw can be motor driven for automatic recording. The worm shaft bearing is pivoted, so that the worm can be engaged or disengaged from its wheel while turning; the clicking spring can also be pushed to one side at will.

An ordinary camera shutter F is mounted above the lamp; it is set permanently to "time exposure," and its bulb release gives a convenient control for taking zero readings.

In order to minimize zero drift it is desirable that the image of the lamp filament be reduced to such size that it just fills the slit. A magnification of 4 was found most convenient for the microscope objectives, as the shorter the distance from emulsion to lens O_2 the greater are the errors introduced by variations in the thickness of the glass of the plate being measured. With the present instrument these errors are of the third order, however, since the density of a line is primarily compared with that of its background, which is measured very close to it, and only secondarily with the natural fog of the plate. It does militate against its use as an ordinary densitometer, however; i.e., an instrument where patches of fairly uniform density widely separated on the plate are to be compared with each other and with the plate fog. Indeed, it appears that a definite need exists for a separate type of instrument for each purpose. For this reason in the present paper one type has been referred to as a micro-photometer, and the other as a densitometer.

For the very satisfactory performance of the instrument described the laboratory is indebted largely to the skill and interest of Mr. S. W. Borrow, instrument maker, who carried out its construction.

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SEPT. 22, 1927.

IMPROVED FORM OF GAS TYPE X-RAY TUBE*

By L. F. Curtiss

Work now in progress required the development of an x-ray tube capable of producing a strong beam of comparatively soft radiation. The gas tube as developed by Müller[1] was naturally considered and some time has been spent in trying several designs. The results have been so successful that a brief description of the final design is given here in sufficient detail to enable others who may have similar requirements to make use of this form of tube.

It may be recalled that this type of x-ray tube has the advantages that no hot cathode is required; that, with proper control of the pressure such as is readily secured under laboratory conditions, it is extraordinarily efficient; and that it is very easy to arrange the parts of the tube so that one may work with the beam within a few millimeters of the target and thus utilize, by virtue of the inverse square law, the increased intensity at this point.

The improvements which the writer has made in the tube as described by Müller is to substitute air-cooling for water-cooling of the cathode and to design a more compact and simple water-jacketed anticathode so that only a single waxed joint is required. The air-cooling of the cathode is made possible by using a copper-glass seal to join the cathode to the pyrex glass forming the body of the tube. In the accompanying figure, which is drawn carefully to scale, the copper fin-radiator R is shown mounted on a short thick copper rod which in turn is soldered into the copper tube sealed at S to the pyrex. This copper rod is screwed to a thick aluminum rod which carries the aluminum cathode C, and serves to conduct the heat rapidly to the radiator. This cathode is sufficiently concave to focus the cathode ray beam. The curvature of the cathode is not very critical. The writer finds that the target should be approximately at the center of curvature but the focusing produced along the beam itself by the presence of gas in the tube reduces the beam to a very small cross section and this fact partially accounts for the very high efficiency of the tube. The lower end of the pyrex tube is closed by a flanged copper bushing, which is hollow to allow water

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[1] Müller, Phil. Mag., *43*. p. 419; 1921.

cooling, and is sealed in position by Khotinsky cement. The bushing has a central tubular extension at the bottom of which is placed the target T. This may be of any convenient material. Below the target is another cavity in which water may circulate and which is connected by a metal tube with the water-jacket in the upper part of the bushing. This makes possible a continuous circulation in the two water jackets with a single inlet and outlet. Opposite the target is a window W

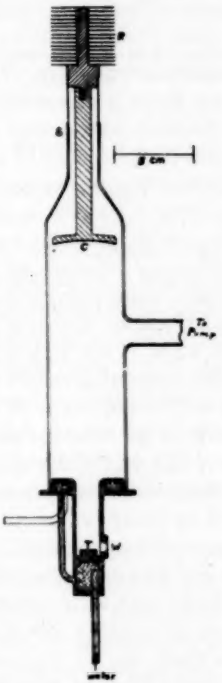

FIG. 1. *Gas type x-ray tube with air-cooled cathode.*

through which the radiation escapes. This window may be covered with thin mica for most purposes. It may be waxed on with Khotinsky as the water-cooling in the arrangement described is quite sufficent to prevent the wax from softening even during prolonged operation.

The tube is best controlled by attaching it to a diffusion pump and introducing an adjustable leak to give the proper pressure. At 20,000 volts the tube works well between 0.5 and 1.5×10^{-3}mm, giving best results at about 0.9×10^{-3}mm. A convenient form of leak is a tapered

needle valve as described by Kaye.[2] Such arrangement enables one to take care of variations in the pressure which might be caused by changes in pumping speed or small leaks. Of course it is advisable to insert a kenotron in series with the tube to cut out the inverse current.

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[2] Kaye, High Vacua, p. 51.

Further advances in mass spectroscopy. The new mass-spectrograph designed and now in use by Aston yields data superior by one order of magnitude to those which the prior instruments gave, the mass of an atom being now determinable (relatively to that of some standard atom such as oxygen) to 1 part in 10,000, while adjacent lines of the mass-spectrum are separated if the masses differ by 1 part in 650. With these powers a tremendous advance in knowledge of exact atom-masses—expressed as "packing-fractions," the difference between an atom-mass (on the scale O = 16) and the nearest integer ("mass-number") divided by that integer—is possible and is in part already made. The data given in Aston's Bakerian lecture relate to H, He, Li, B, C, N, F, Ne, P, S, Cl, A, As, Br, Ks, Sn, I, Xe, and Hg. Among the specially interesting results are these: ratio of masses H to O is 1.00778:16, of He to O is 4.00216 to 16, of C to O is 12.0036 to 16 ("no particular significance is to be attached to the final digit"); F and the predominant isotopes of Ne and S have practically integer mass-numbers; tin has no fewer than eleven isotopes, four of which are odd and one of these four an isobar of an isotope of another element (both of these last facts are exceptions to earlier-announced rules); the packing fractions of the isotopes of Xe and Sn are sensibly identical (in contradiction with a prior result). No data could be obtained with W, a disappointment. An interesting graph of packing fractions vs. mass-number M shows a fairly smooth curve which lies on the positive side of the axis for $M < 20$, descends to the negative side and rises slowly again after a minimum near $M < 60$ and recrosses the axis near $M = 190$. Below $M = 16$ the curve bifurcates, the branch for odd atomic numbers lying higher than that for even atomic numbers. [F. W. Aston, (Cavendish); Proc. Roy. Soc., *A115*, pp. 487–514; 1927.]

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